1 An Illustrative Model

In the following partial equilibrium model firms produce multiple output types – goods and services – and must decide how to allocate their accumulated expertise, or knowledge, across the production of each. We take the level of expertise as exogenous in the model and explore its content in the empirics. The scarce nature of the expertise, and its confinement to the firm, induces a tradeoff in goods and services production and generates predictions regarding how firms adjust production in the face of changing market conditions, such as lower manufacturing import tariffs.¹

Demand

We consider a multi-country partial-equilibrium setting. In each country, there is a continuum of industries in which a representative agent consumes industry-specific goods and services. The agents’ preferences over total industry output are Cobb-Douglas everywhere such that the share of aggregate expenditure spent on industry $j$ is $\kappa_j$, where $\int_0^1 \kappa_j dj = 1$. Furthermore, the share of industry $j$ expenditure that is spent on services output from

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¹An alternative framework is that of Bloom et al. (2012) in which firms reallocate production factors in “bad times” when the opportunity cost of doing so is relatively low. Different from that paper, here we focus on the long run while explicitly modeling the degree of rivalry in the use of inputs across different types of production.
that industry is $\nu_j$. We therefore denote by $E_{jS} \equiv \kappa_j \nu_j E$ and $E_{jG} \equiv \kappa_j (1 - \nu_j) E$ the expenditure on services and goods output, respectively, from industry $j$, where $E$ is total expenditure in the economy.

We assume that preferences for goods and services are separable and within an industry are given by independent Constant Elasticity of Substitution (CES) utility functions. There is a large number of firms active in each industry and each firm provides a single differentiated good and services variety. Firms are monopolistically competitive and ignore the impact of their choices on aggregate quantities when setting prices. The CES demand for the variety of good and the variety of service produced by firm $i$ in industry $j$ from country $n$ can be written separately as:

$$q_{ijnG} = p_{ijnG}^\sigma P_{jnG}^\sigma E_{jnG}$$

$$q_{ijnS} = p_{ijnS}^\gamma P_{jnS}^\gamma E_{jnS}$$

where $\sigma > 1$ denotes the elasticity of substitution across varieties of goods and $\gamma > 1$ denotes the elasticity of substitution across services varieties. The industry price indices in country $n$ can be written as

$$P_{jnG} = \left(\int_{\omega_{nG}} \left[p_G (\omega_G)\right]^{1-\sigma} d\omega_G + \int_{\omega_{nG}^*} \left[p_G^* (\omega_G^*)\right]^{1-\sigma} d\omega_G^*\right)^{\frac{1}{1-\sigma}}$$

$$P_{jnS} = \left(\int_{\omega_{nS}} \left[p_S (\omega_S)\right]^{1-\gamma} d\omega_S + \int_{\omega_{nS}^*} \left[p_S^* (\omega_S^*)\right]^{1-\gamma} d\omega_S^*\right)^{\frac{1}{1-\gamma}}$$

and $\Omega_G$ and $\Omega_S$ denote, respectively, the set of services and goods varieties available from home producers in country $n$, while $\Omega_{nG}^*$ and $\Omega_{nS}^*$ denote the sets of foreign varieties. In the following, we take conditions on all markets (i.e., $P_{jnG}$, $P_{jnS}$, $E_{jnG}$, and $E_{jnS}$) as exogenous and explore firm production choices in response to changes in these conditions. In the empirics, we will control for market conditions through appropriate proxy variables and fixed-effect combinations. For ease of notation, we drop industry subscripts $j$ from now on.

**Production**

We assume that firm $i$’s production functions for goods and services take the following form:

$$Y_{iG} = \Lambda_{iG} T_{iG} L_{iG}$$

$$Y_{iS} = \Lambda_{iS} T_{iS} L_{iS}$$

where $\Lambda_{il}T_{il}$ is a firm-specific productivity term that is comprised of a fixed, exogenously determined component, $\Lambda_{il}$, and an endogenously chosen component, $T_{il}$, where $l \in (G, S)$. The firm’s labor input is $L_{il}$.

One of the key features of the model is our interpretation of $T_{il}$ which, motivated by the stylized facts and discussion above, we assume to reflect the extent to which the firm’s accumulated industry-specific expertise is directed toward one output type or the other. Over time firms both passively and actively accumulate knowledge (expertise) about the
products they are selling and the markets they are selling to. Since this knowledge is, to some extent, embodied in workers and managers whose time is limited, it must be apportioned efficiently within the firm. This is a notion that the business literature has consistently found evidence for.\(^2\)

Formally, we assume that the stock of expertise is both fixed within the firm and rivalrous in its use across output types in the sense that increased use of expertise in producing one output type reduces the expertise available in producing the other output type. We model the degree of rivalry in expertise across goods and services production in the following reduced-form way:

\[
T_i = \left( (T_i^G)^t + (T_i^S)^t \right)^{1/t}
\]

where we assume that \( t \in (0, \infty) \) governs the extent of rivalry in the use of expertise across output types. Note that a higher \( t \) implies less rivalry: for \( t \to \infty \), firms can use the full amount of \( T_i \) in both goods and services production.

We assume that firms exporting to foreign destinations face standard variable iceberg-type trade costs in goods and services, denoted by \( \tau^G_j \) and \( \tau^S_j \), respectively. Given this setup, the profit maximization problem of firm \( i \) selling to \( N \) markets is:

\[
\max_{p_{iG}, p_{iS}, T_i^G, T_i^S} \pi_i = \sum_{n=1}^{N} [p_{iG}Y_{inG} + p_{iS}Y_{inS} - w_i (L_{inG} + L_{inS})]
\]

s.t. \[ T_i = \left( (T_i^G)^t + (T_i^S)^t \right)^{1/t} \]

where \( p_{iG} \) and \( p_{iS} \) are price vectors containing the prices charged in each destination market (including the firm’s home market), and \( L_{inG} = \tau^G_n Y_{inG}/\Lambda_{iG} T_i^G \) and \( L_{inS} = \tau^S_n Y_{inS}/\Lambda_{iS} T_i^S \) are the amounts of labor required to deliver \( Y_{inG} \) and \( Y_{inS} \) units of goods and services to country \( n \), respectively.

Substituting in (1), (2), and (5), firm profit maximization can be written as:

\[
\max_{p_{iG}, p_{iS}, T_i^G} \pi_i = \sum_{n=1}^{N} \left( p_{iG}^{\sigma-\gamma} p_{iS}^{\sigma-\gamma} \frac{E_{nG}}{\Lambda_{iG} T_i^G} + p_{iS}^{1-\gamma} \frac{E_{nS}}{\Lambda_{iS} ((T_i)^t - (T_i^G)^t)^{1/t}} \right) - w_i \sum_{n=1}^{N} \left( \frac{\tau^G_n p_{iG}^{\gamma} p_{iG}^{\gamma-1} E_{nG}}{\Lambda_{iG} T_i^G} + \frac{\tau^S_n p_{iS}^{\gamma} p_{iS}^{\gamma-1} E_{nS}}{\Lambda_{iS} ((T_i)^t - (T_i^G)^t)^{1/t}} \right)
\]

The firm’s optimal prices for each industry in each destination is then given by:

\[
p_{iG} = \frac{\sigma}{\sigma - 1} \frac{\tau^G_i w_i}{\Lambda_{iG} T_i^G}
\]

\(^2\)For instance, Visnjic and Van Looy (2009) summarize the accepted view as follows: “When a firm starts to provide services...there is a natural knowledge relatedness to be exploited on the level of technological capabilities and knowhow that can be transferred from product engineering departments to the service activities of the firm...Technological expertise represents assets that can be leveraged when engaging in service activities.”
\[ p_{inS} = \frac{\gamma}{\gamma - 1} \frac{\tau_n^S w_i}{\Lambda_{is}(T_i)^t - (T_{iG})^t}^{1/t} \] (7)

The firm faces a clear tradeoff. By directing more expertise toward goods production (i.e., increasing \( T_{iG} \)) the firm is able to lower its output price for goods and improve its competitiveness in the goods market at the expense of services production. Ultimately, the firm’s optimal allocation will depend on the relative marginal profitability of goods versus services across all markets. Solving for this optimal allocation decision, and substituting in the optimal prices (6) and (7), the equilibrium expertise directed toward goods production can be written (services is symmetric):

\[ T_{iG} = \frac{\sigma - \gamma}{\sigma - 1} \left( \frac{T_i}{T_{iG}} \right)^t \left( 1 - \frac{1}{\sigma} \right) = \frac{\sigma - \gamma}{\gamma - 1} \mu_{is} RMC_i \] (8)

where \( \mu_{is} \equiv \left( \frac{\gamma}{\gamma - 1} \right) \left( \frac{\tau_n^S w_i}{\Lambda_{is}} \right) \), \( \mu_{ig} \equiv \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{w_i}{\Lambda_{ig}} \right) \), and \( RMC_i \equiv \frac{\sum_{n=1}^{N} \left( \frac{\tau_n^G w_i}{\Lambda_{ig}} \right) \left( \frac{\tau_n^S w_i}{\Lambda_{is}} \right)^{1-\sigma}}{\sum_{n=1}^{N} \left( \frac{\tau_n^G w_i}{\Lambda_{ig}} \right) \left( \frac{\tau_n^S w_i}{\Lambda_{is}} \right)^{1-\gamma} E_{nG}} \sum_{n=1}^{N} \left( \frac{\tau_n^G w_i}{\Lambda_{ig}} \right) \left( \frac{\tau_n^S w_i}{\Lambda_{is}} \right)^{1-\sigma} E_{nS} \)

summarizes the “relative market conditions” faced by firm \( i \), i.e., the relative residual demand for its goods and services in all locations. The allocation decision is therefore a function of relative market conditions (RMC), the firm’s aggregate stock of expertise (\( T_i \)), the elasticity parameters associated with goods and services markets (\( \sigma, \gamma \)), and the degree of rivalry in the use of expertise within the firm (\( t \)).

We can also derive the goods and services revenues that the firm receives in each market in this partial equilibrium, as:

\[ R_{inG} = \left( \frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \left( \frac{\tau_n^G w_i}{\Lambda_{ig} T_{iG}} \right)^{1-\sigma} (P_{nG})^\sigma E_{nG} \] (9)

\[ R_{inS} = \left( \frac{\gamma}{\gamma - 1} \right)^{1-\gamma} \left( \frac{\tau_n^S w_i}{\Lambda_{is} T_{iS}} \right)^{1-\gamma} (P_{nS})^\gamma E_{nS} \] (10)

where the optimal allocation of \( T_{iS} \) and \( T_{iG} \) is given by (8) and its services counterpart.

**Comparative Statics**

The focus of the empirics will be on the extent to which firms alter their production strategy in the face of trade liberalization (i.e., in the face of lower tariffs on goods imports). In the model, a decline in domestic import tariffs leads to a fall in the goods price index at home (\( P_{HG} \)), and thus a corresponding decline in the domestic residual demand for goods. Reiterating the results from above, condition (8) indicates that the firm’s response will depend on its aggregate stock of expertise (\( T_i \)), the extent to which expertise is “freely available” within the firm (governed by \( t \)), and the demand elasticities...
σ and γ.

The result is an ambiguous response on the part of firms to lower import tariffs. To see this, we can differentiate the equilibrium condition (8) with respect to the domestic goods price index, $P_{HG}$. This leads to sufficient conditions under which the firm will respond by reallocating expertise toward services provision. The flip side are conditions under which the firm will respond by increasing the expertise allocated to goods production.

**Proposition 1 – Fight:** Firms will “fight” following a decline in domestic goods import tariffs, $\frac{\partial T_i}{\partial P_{HG}} < 0$, when:

\[(\gamma - \sigma) \left(\frac{T_{iG}}{T_i}\right)^t > \gamma(1-t) - \sigma + t(1 + t)\]

That is, when the price index in the domestic goods market falls, firms reallocate $T_i$ from provision of services to production of goods. The above will hold for all firms when $1 + t < \gamma < \sigma$.

*Proof is relegated to the appendix ■*

Recall that expertise serves to enhance productivity, such that by choosing the allocation of expertise the firm is in effect choosing its relative productivity across output types. When the goods elasticity ($\sigma$) is large relative to the services elasticity ($\gamma$), the marginal increase in profits associated with a marginal reallocation of expertise toward goods production exceeds the increase from allocating additional expertise toward services provision. Thus, the firm will shift $T_i$ from services to goods in order to lower the goods price and remain viable in that market.

In addition, from (5) we can see that for a given stock of expertise, $T_i$, both $T_{iG}$ and $T_{iS}$ decrease as $t$ falls. In effect, this is because for smaller $t$ (more rivalrous expertise) there is less “shared” expertise across output types. As a result, a further implication of Proposition 1 is that expertise must be sufficiently rival in order for reallocation to be efficient – i.e., $t$ must be sufficiently small for firms to remove resources from services in order to maintain standing in the goods market. In this case, firms reinforce their position in the goods market in order to mitigate a potentially severe loss in market share, but must remove resources from services to do so since knowledge is relatively non-transferrable.

We believe, and our empirics will support, a more intuitive scenario where firms flee from competition.

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3In our partial equilibrium framework, $P_{HG}$ and its components are taken as exogenous so that we can take derivatives with respect to $P_{HG}$. Differentiating with respect to $P_{HG}$ is equivalent to differentiating with respect to domestic import tariffs in this setting (see below).
Proposition 2 – **Flee**: Firms will “flee” following a decline in goods import tariffs, \( \frac{\partial T_iG}{\partial \tau_{HG}} > 0 \), when:

\[
(\gamma - \sigma) \left( \frac{T_iS}{T_i} \right)^t > t(\gamma - 1 - t).
\]

That is, when the price index in the domestic goods market falls, firms reallocate \( T_i \) from production of goods to provision of services. The above will hold for all firms when, \( \sigma < \gamma < 1 + t \).

**Proof is relegated to the appendix.**

Now, a large \( t \), reflecting less rivalrous expertise within the firm, makes it more likely that firms flee from competition. In this case, firms have more resources simultaneously available to both output types and can therefore shift production toward the relatively less competitive services sector with only a relatively small loss in market share in the goods market.

In short, firms face a flee or fight decision which turns on the relative price elasticities of the two markets and the degree of rivalry of firm-specific expertise. Since the empirics will exploit reductions in import tariffs (\( \tau^{G}_H \)) as a source of trade liberalization, it is worth being explicit about the role of tariffs in the model. Propositions 1 and 2 imply the following:

**Corollary 1.1** When Proposition 1 holds, \( \frac{\partial T_iG}{\partial \tau_{HG}} < 0 \). When Proposition 2 holds \( \frac{\partial T_iG}{\partial \tau_{HG}} > 0 \).

These conditions follow directly from the positive relationship between the price indices and import tariffs. The empirics will provide a framework test these predictions.

Finally, for a given value of the rivalry parameter, \( t \), the size of the aggregate stock of expertise matters for firm adjustment. Formally:

**Proposition 3** Given equilibrium condition (8) the sign of \( \frac{\partial^2 T_iG}{\partial \tau_{HG} \partial T_i} \) will be the same as the sign of \( \frac{\partial T_iG}{\partial \tau_{HG}} \), as long as the elasticity of expertise in services with respect to total expertise is greater than unity, \( \frac{\partial T_i}{\partial T_i} \frac{T_i}{T_S} > 1 \).

**Proof is relegated to the appendix.**

Consider the case in which firms flee (i.e., \( \frac{\partial T_iG}{\partial \tau_{HG}} > 0 \)). Proposition 3 states that the extent to which a firm flees is heterogeneous across firms, and is a function of the firm’s stock of expertise – i.e., firms with a relatively large stock of expertise will shift relatively more into services in response to trade liberalization.

To summarize, we motivated the structure of our model in large part by pointing to the reduction in UK manufacturing import tariffs and the simultaneous growth of services sales by UK manufacturing firms relative to their goods sales. In addition,
we found a strong negative correlation between goods and services revenues within UK firms, suggesting a tradeoff in production over the period. The structure of our model led straightforwardly to Propositions 1 and 2, and Corollary 1, which indicate that it is unclear whether firms will flee or fight when faced with trade liberalization, with the response depending on demand conditions in the two sectors and the degree of rivalry in the use of firm-level expertise. Finally, Proposition 3 indicates that having a larger stock of expertise magnifies the extent of reallocation when trade liberalizes, whatever its direction. We next describe the data we use to determine and evaluate the empirically relevant cases.

A  Proof of Propositions

Proof of Propositions 1 and 2

We begin by totally differentiating (8) with respect to the goods price index, \( P_G \). This yields:

\[
\frac{\partial T_G}{\partial P_G} = \frac{\partial RMC_i}{\partial P_G} \frac{T_G}{RMC_i} \Omega \tag{11}
\]

where \( \Omega \equiv \sigma \gamma - (\gamma - 1 - t) \left( \frac{T_G}{T_S} \right)^t \).

The sign is therefore determined by the ambiguous term, \( \Omega \), that takes into account the relative use of \( T \) in each output type and its relation to the elasticities of substitution in each sector. The sufficient conditions in Proposition 1 can be derived simply by noting that \( \Omega \) will be positive when both \( \sigma > \gamma \) and \( \gamma > 1 + t \). Similarly, it will be negative under the reverse conditions. \( \blacksquare \)

Proof of Proposition 3

Differentiating (11) with respect to \( T \) yields:

\[
\frac{\partial RMC_i}{\partial P_{HG}} \left( \frac{\partial T_G}{\partial T} - \frac{t(\gamma - 1 - t) T_G}{T_S} \left( \frac{T_G}{T_S} \right)^{t-1} \left( 1 - \frac{T}{T_S} \frac{\partial T_S}{\partial T} \right) \right) \Omega
\]

where \( \Omega \) is defined as above. The sign of this derivative depends once again on the relative values of the substitution parameters \( (\gamma, \sigma, \text{ and } t) \). However, under the sufficient conditions from Propositions 1 and 2, we can pin down the direction of the second derivative. We have two cases:

1. When \( 1 + t < \gamma < \sigma \), Proposition 1 holds since \( \Omega > 0 \). Since \( \frac{\partial T_G}{\partial T} > 0 \), \( \frac{\partial^2 T_G}{\partial P_{HG} \partial T} \) will be the same sign as \( \frac{\partial T_G}{\partial P_{HG}} \) when \( 1 - \frac{T}{T_S} \frac{\partial T_S}{\partial T} < 0 \).
2. When \( \sigma < \gamma < 1 + t \), Proposition 2 holds since \( \Omega < 0 \). Again, since \( \frac{\partial T_G}{\partial T} > 0 \), \( \frac{\partial^2 T_G}{\partial P_H G \partial T} \) will be the same sign as \( \frac{\partial T_G}{\partial P_H G} \) when \( 1 - \frac{T}{T_S} \frac{\partial T_S}{\partial T} < 0 \).

References
