

# Technological Change and Within-Occupation Inequality

Tianxi Wang and Greg C. Wright

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**Abstract:** Over the period 2003 to 2010, more than half of overall growth in U.S. income inequality occurred *within* occupations. We focus on two types of technological change as potential determinants of this trend, both of which increase income inequality within occupations in which workers perform similar tasks but produce differentiated work output. The first type of technological change, reflected in the recent spread of information and communication technologies (ICT), enables a given quantity of worker output to be consumed or used to a greater extent. We model this as an increase in the limit up to which an occupation's production technology displays increasing returns to scale. This drives the workers with the least human capital out of the occupation and alters the incomes of the remaining workers such that the higher their initial income, the more they gain (or the less they lose), which leads to rising inequality. Second, we show that via a novel general equilibrium effect, *any* technological change that raises aggregate income can also widen income inequality. Hence, increased automation in the oil industry can contribute to a divergence of incomes among musicians. We compare the theoretical results with empirical estimates derived from the natural experiment provided by the advent and spread of the internet, finding support for the predictions.

JEL: J24, J31, O30, D33

Keywords: income inequality, technological change, increasing returns to scale to some limit

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<sup>1</sup>Tianxi Wang: Department of Economics, University of Essex. Email: wangt@essex.ac.uk. Tel: +44 (0)1206873480. Corresponding Author: Greg Wright: University of California, Merced, 5200 N Lake Rd., Merced, 95343. Email: gwright4@ucmerced.edu.

## 1. Introduction

We introduce two new theoretical channels through which technological change may increase within-occupation income inequality. In both cases, the way in which tasks are performed within an occupation remains unchanged, and yet the income distribution within the occupation widens as the result of the technological change. The fact of widespread growth in income inequality has been a key focus of much of the economics literature in recent years, and it has more recently been observed that a substantial portion of the increase has occurred *within* occupations (we review this literature in Section 2). For instance, growth in within-occupation inequality between 2003 and 2010 explains 54 percent of the overall growth in U.S. inequality over the period, clearly an important amount (see Appendix C for details).<sup>1</sup> However, for certain individual occupations it is not clear exactly how, and to what extent, technological change has played a role, even within occupations that have experienced vast increases in inequality. Consider, for example, professional football (soccer) players in England, who compete in one of several leagues, where the Premier League is the highest-level league, League Two is the lowest, and the Championship and League One are in-between. As Figure 1 shows, the average income of League Two footballers has changed very little since 1992 and, in fact, closely tracks average U.K. income over the period, whereas the average income of Championship players diverges from the national average only modestly. In contrast, Premier League footballers have seen explosive growth in average income over the period, which has drastically diverged from both lower-league incomes as well as the U.K. average.

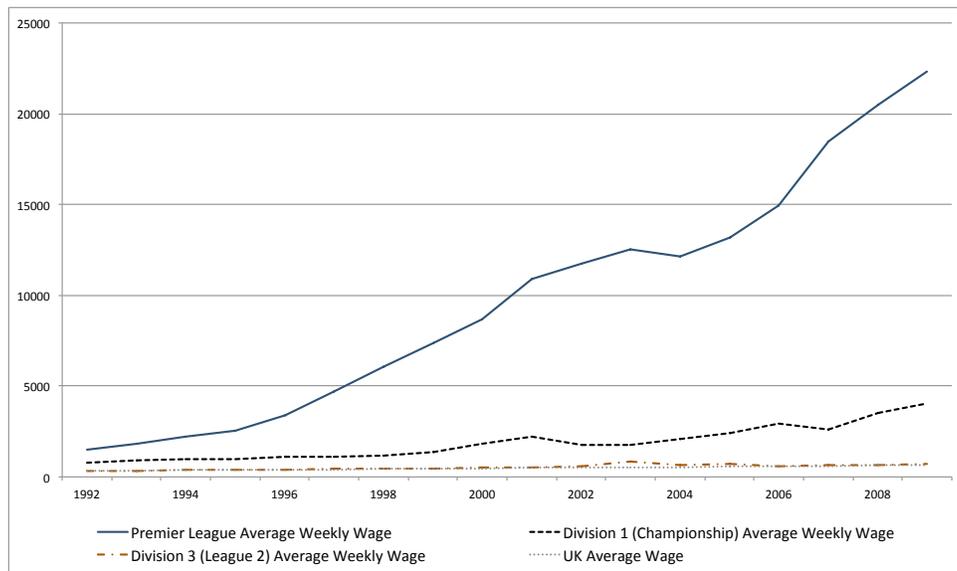


Figure 1: Wages Across UK Football Leagues & Average UK Wage, 1992-2010. Source: Mail on Sunday

<sup>1</sup>A significant change in U.S. occupation codes in 2003 leads us to limit the analysis to this particular time period.

The example of footballers highlights the challenge that economists face when considering the role played by technological change in driving within-occupation inequality. In particular, the dominant model of technological change, that of Skill-Biased Technical Change (SBTC), is not very useful in this case since it is difficult to argue that new technologies are neutral with respect to the tasks performed by footballers in League Two and yet simultaneously complement the tasks performed by Premier League footballers. More to the point, it is clear that new technologies have not, in fact, altered the performance of footballing tasks at all.

In this paper we focus on two channels through which technological change can increase income inequality within an occupation, without altering the performance of the occupation's tasks. The first is by increasing the scale of operation of workers, defined as the extent to which the worker's output is used or consumed. The second arises due to any increase in labor productivity, and therefore aggregate income. This channel increases inequality within occupations in which workers perform similar tasks but produce differentiated output, much like footballers.

The first channel is motivated by the observation that new Information and Communication Technologies (ICT) often increase the scale of operation for workers within certain occupations. For example, the advent of television and, more recently, the internet has vastly expanded the potential number of viewers for football games – i.e., these technologies have vastly expanded the scale of operation for footballers. Of course the extent to which workers can reach consumers and users has always been important, as evidenced by the vast changes in the market for music brought about by the advent of the radio in the early 20th century. Within the economics literature the importance of workers' scale of operation has not gone unnoticed: a strand of the literature beginning with Rosen (1981) has applied this notion to explain certain features of the income distribution. However, as we describe in Section 2, this literature has not formally analyzed the effects of an increase in the scale of operation, as we do here.

Our modeling approach rests on a key observation: for occupations in which the scale of operation is an important determinant of workers' income, the production technology necessarily displays Increasing Returns to Scale *up to some Limit* (which we denote IRSL). This limit defines the scale of operation for an occupation and, taking the limit to infinity, IRSL subsumes IRS as a special case. To be more specific, the IRS component of IRSL is typically driven by the fact that once a service has been produced, at some fixed cost, it can be simultaneously consumed or used by many individuals without much degradation in its effect; that is, it can create value at constant returns to scale (CRS). Our innovation consists of the observation that the CRS often operate up to some limit. For example, it is costly to create a live music performance, but it costs little to admit an additional person into the theater to hear it up to the point that the theater is filled; the capacity of the theater thus defines the limit of IRS for the musicians. Similarly, while it may be costly for the manager of a firm to identify a profitable strategy, once one has been identified the profit it

generates may rise in proportion to the resources that are deployed – i.e., at CRS – until all the firm’s resources have been expended. The scale of these resources, i.e., the firm’s size, is thus the limit of IRS for the manager’s job.

Our model captures this observation. In the model, we consider a continuum of agents with equal endowments of unskilled labor but heterogeneous endowments of human capital. They choose to subsist either by employing their labor, or by employing their human capital and thereby becoming a “professional”, for which the quality of their output depends on the size of their human capital endowment.<sup>2</sup> Professionals hire labor in order to produce a stream of services and labor is also separately used to produce a subsistence good. Labor is homogenous and its providers compete under perfect competition, whereas the services provided by individual professionals are differentiated. In general, this is because labor (i.e., the human body) is a relatively homogenous input: in most instances one individual’s labor can substitute for another’s. On the other hand, human capital (i.e., the human mind) is much more idiosyncratic and diverse. As a result, professionals compete via monopolistic competition. We assume that after committing her time to supply human capital rather than labor, a professional hires labor to produce her variety of services at constant returns to scale up to the technological limit, denoted  $B$ . This  $B$  therefore represents the maximum scale of operation for a professional – for example, the capacity of the theater in which a musician performs. Technological progress that increases  $B$  thus increases the maximum, *potential* scale of operation for all professionals.<sup>3</sup> On the other hand, the productivity of labor, measured by the quantity of the subsistence good produced per unit of labor, is denoted by  $A$ . We model technological progress as an increase in either  $B$  or  $A$ , the implications of which for the income distribution and career choice are the focus of this paper.

We show that an increase in either  $B$  or  $A$  will widen inequality between professionals within an occupation, but via very different mechanisms. Consider first an increase in  $B$ , the IRS limit. On the one hand, each professional within the occupation can sell more, which benefits them. On the other hand, since the capacity of all professionals increases, each of them faces fiercer labor market competition. Whereas the increase in competition is the same for all professionals, the expansion in capacity delivers greater benefits to those who have more human capital and are therefore able to charge a higher price for their stream of services. As a result, more talented professionals reap greater gains from a given increase in  $B$ , which therefore raises income inequality between professionals. Now consider an increase in  $A$ . This increases the productivity of labor hired by professionals as well as the wage paid to labor, which results in no net effect on the marginal cost, nor the profit,

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<sup>2</sup>This feature of an identical labor endowment and heterogeneous human capital (or ability) is also found in Lucas Jr (1978) and Monte (2011).

<sup>3</sup>However, this does not necessarily increase the *actual* scale of operation for a particular professional. For example, the internet may enable a singer to access several hundred million potential customers, but the singer will likely be unable to sell to each of these people.

associated with professionals' production of their stream of occupational services. Nevertheless, an increase in  $A$  widens income inequality between professionals due to a general equilibrium effect, with the intuition as follows. First, the primary impact of an increase in  $A$  is to raise aggregate income. Second, since professionals with greater human capital acquire a larger fraction of aggregate income by providing better quality services, a larger fraction of the increase in aggregate income accrues to them. This again increases income inequality.

Returning to the example of footballers, the two channels are as follows. First, innovations in ICT have increased the size of the potential audience for football matches. This has intensified the competition between the football leagues for viewers, which has ultimately shifted viewers away from the lower leagues in favor of the Premier League. Second, labor productivity increased over the period, raising the overall income of the U.K. public. As a result, consumers were willing to pay more for football entertainment, an effect that is larger for footballers in higher-quality leagues.<sup>4</sup> And finally, footballers in League Two, the lowest league, can be thought of as the marginal professionals in our model who, in equilibrium, are indifferent between becoming footballers or finding other work. As a result, our paper predicts that their earnings will track average income in the U.K. economy, which has in fact been the case.

While an increase in either  $B$  or  $A$  will raise income inequality, the consequences for the career choice of each agent are quite different. First, an increase in  $A$  induces more agents to exploit their human capital relative to their labor, leading to a greater variety of differentiated goods. This effect is consistent with the long-run historical shift away from labor-intensive work and toward human-capital-intensive work, such as the growth in the supply of artists, writers, sportsmen, comedians, etc. On the other hand, an increase in  $B$  drives agents *out of* human-capital-intensive work by intensifying competition between professionals. This result is consistent with a range of anecdotal observations on the effects of ICT advancement. For example, the popularity of television makes it difficult to earn a living as a local comedian, while the expansion of the internet reduces opportunities for travel agents.

In a final section we discuss the extent to which the model is consistent with the empirical evidence. We explore expansions in market access (increases in the IRS limit,  $B$ ) arising from recent technological change using U.S. data, where we exploit the natural experiment generated by the rapid expansion of global internet access beginning in the mid-1990s. Specifically, we test the predictions of an increase in  $B$  on within-occupation inequality and career choice for the occupations for which the IRS limit has increased due to the internet. To identify these affected occupations, we construct a measure of the extent to which the stream of services associated with

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<sup>4</sup>In fact, Chinese football leagues have recently begun signing high-quality European and Latin American players, a trend that is almost certainly in large part due to rising incomes, and therefore rising demand for entertainment, in China.

each U.S. occupation generated internet sales in each year over the period 1995-2006. An increase in this measure for a particular occupation therefore reflects an increase in the reach or scale of operation for those workers – i.e., an increase in the IRS limit. In a regression framework we then test the relationship between this measure and changes in wage inequality and employment across occupations, controlling for pre-internet trends and potentially confounding variables. Noting that our analysis should be interpreted as suggestive rather than definitive, we show that the results are consistent with the theoretical predictions of the model.

The paper proceeds as follows. Section 2 places our paper within the existing literature. Sections 3 and 4 present our theory of technological change. Section 5 brings some predictions of the model to the data. Section 6 provides concluding remarks.

## 2. The Literature

The general theme of our paper, namely that technological change may increase income inequality, fits within a large strand of literature that approaches the topic from a variety of perspectives. However, the dominant theoretical argument is the theory of Skill-Biased Technical Change (SBTC).<sup>5</sup> Relative to SBTC and its extensions, our paper focuses on within-occupation inequality while also considering different facets of technological progress, which we believe complements the existing literature.

Specifically, our contribution is twofold. First, we adopt the notion of IRSL to model an important consequence of ICT, namely that it increases workers' access to markets. This approach represents a new way of modeling technology relative to the conventional approach which models technology as a labor- (or capital-) augmenting factor. With this modeling approach in hand, we uncover a new mechanism through which technological changes affect the income distribution, while also deriving new empirical implications. As noted, technological change which increases the IRS limit alters the income distribution through competition and workforce reallocation, a mechanism that has little in common with the standard SBTC model. As a result, unlike SBTC, technological change that increases the IRS limit differentially affects workers who are engaged in *the same type of work* (e.g., singers, footballers or eBay merchants).<sup>6</sup> This is important in light of recent

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<sup>5</sup>As just a few representative examples: Tinbergen (1974) is an example of early work linking the demand for skill to technology; Autor, Levy and Murnane (2003) find that computers displace routine workplace tasks and complement cognitive-intensive, non-routine tasks; Firpo, Fortin and Lemieux (2012) find an important role for technology in generating the observed inequality pattern over the 1980s, 1990s, and 2000s; Beaudry, Doms and Lewis (2010) find that computer adoption increases the return to skill; and Chen, Forster and Llana-Nozal (2013) find that technology has increased inequality across OECD countries. Recently, Acemoglu and Autor (2011) have extended the standard SBTC framework to endogenize the matching of skills to tasks.

<sup>6</sup>The differential effects in our paper arise from differences in workers' human capital endowments, similar to the literature that is built on the matching model of Sattinger (1993). Also, in a recent paper Guvenen and Kuruscu (2012) show that simply adding heterogeneity in the ability to accumulate human capital to a standard SBTC framework generates a set of predictions for wages that matches features of the U.S. wage distribution over the past four decades

empirical evidence that within-occupation variation in income is important, for example see Kim and Sakamoto (2008) for the U.S. and Helpman, Itskhoki, Muendler and Redding (2012) for Brazil, as well as our calculations for the U.S. reported above (for details see Appendix C).

Second, we show that any technological change that increases aggregate income can contribute to a rise in income inequality between workers who provide differentiated products or services. This is due to a general equilibrium effect that, so far, has been missing in the literature. According to this result, technological change that increases productivity in the natural gas sector can generate a divergence in incomes between singers, or workers within another occupation that is entirely unrelated to natural gas production.

The role of scale of operation as it relates to the return to skill has long been noted in a strand of the literature that seeks to explain certain features of the top of the income distribution, i.e., the earnings of “superstars”; for example, see Rosen (1981), Rosen (1983), Gabaix and Landier (2006), and Egger and Kreickemeier (2012), and see Neal and Rosen (2000) for a summary. However, this literature is mainly concerned with the level of income inequality for a *given* level of technology, and in particular with explaining how small differences in talent can lead to large differences in income. At the same time, it offers only an informal discussion regarding the potential impact of an increase in the scale of operation, whereas we formally model this using the notion of IRSL. In addition, we also study the occupational choice margin, between providing labor and becoming a low-end professional (who are certainly not superstars), whereas this margin is absent in that literature. Lastly, the general equilibrium effect that we introduce is new to the literature.

Our model has some of the flavor of Melitz (2003),<sup>7</sup> though the two papers are concerned with different issues.<sup>8</sup> Both papers feature monopolistic competition with CES preferences, IRS, and agent heterogeneity. However, in our paper IRS operates up to some finite limit, whereas in Melitz (2003) this limit is infinity. As a result, while an increase in  $B$  in our paper might be regarded as parallel to an increase in the number of trading partners in Melitz (2003) (since both reflect an increase in market size), the mechanism whereby this increase drives re-allocation is different. In Melitz (2003), it works through the factor market rather than the product market. In contrast, in our paper, the cost of the factor, which is labor, is unchanged, and the increased competition works through the product market, by lowering the price of the varieties that are pushed out. On the other hand, this effect on the product market is captured by Melitz and Ottaviano (2008) using a

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in a simulation exercise, though there is no attempt to explain growth in within-occupation inequality.

<sup>7</sup>More accurately, our model is in line with Melitz (2003)-style models that incorporate heterogeneity in product quality, since the heterogeneity we introduce augments the marginal value of a unit of consumption, as in those models. For instance, see Baldwin and Harrigan (2007) or Kugler and Verhoogen (2012).

<sup>8</sup>Specifically, our paper is concerned with the effects of technological progress on the income distribution in a closed economy, while Melitz (2003) is focused on the relationship between exporting and aggregate productivity in an open economy.

model of monopolistic competition with quadratic preferences. However, this approach then leads to different implications for changes in market size. Whereas in Melitz and Ottaviano (2008) a larger market supports a greater number of varieties, in our paper an increase in  $B$  reduces the number of varieties, as an increased number of trading partners does in Melitz (2003).

There is other theoretical work that studies the effects of technological progress on the income distribution from a different angle or in a different context. For instance, Garicano and Rossi-Hansberg (2014) extend Lucas Jr (1978) in order to demonstrate the implications of ICT innovation for the income distribution, which they model as a reduction in the rate at which the marginal return to the labor working for a manager falls. This does not intensify competition between managers and no manager loses due to this change. In contrast, an increase in the IRS limit in our model generates effect by increasing competition between workers, which consequently incurs losses for the lowest earners. Other work includes Jones and Kim (2012) who endogenize the Pareto income distribution in a model in which technological progress augments the effects of entrepreneurs' efforts to increase productivity. Garicano and Rossi-Hansberg (2004, 2006) and Saint-Paul (2007) examine the effects of reduced communication costs on the income distribution, where knowledge production and the organization of this production play an important role. Saint-Paul (2006) studies how productivity growth affects income inequality when consumers' utility from product variety is bounded from above.

### 3. The Model

The economy is populated by a continuum of agents. Agent  $i \in [0, 1]$  is endowed with one unit of labor and  $h_i$  units of human capital. Without loss of generality, let  $h_i$  be increasing in  $i$ , that is  $h'_i := \frac{dh_i}{di} \geq 0$ . Agents choose to live on their labor endowment (i.e. muscles) or on their human capital (i.e. brains).<sup>9</sup> In the latter case, they provide a stream of services which, to fix ideas, we assume throughout to be entertainment services (which we previously referred to more generally as professional services). The quality of the services provided by an agent depends on the size of her human capital endowment and, for simplicity, is assumed to be equal to it.

Labor is used for producing both a subsistence good (such as food) and entertainment services. The production of the subsistence good displays constant returns to scale. If  $L$  agents are employed to produce the subsistence good, then its aggregate output is

$$Y = AL.$$

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<sup>9</sup>Of course, in reality nearly all occupations require both muscle and brain. But clearly some occupations demand more human capital relative to labor, while others demand relatively more labor. For simplicity, we abstract from this continuum of human-capital-to-labor ratios, modeling it as a binary choice.

Hence  $A$  represents the productivity of (unskilled) labor.

The production of entertainment services requires two factors, human capital and labor. In this paper, we let human capital affect only the quality of the output and abstract from the effect of human capital on the quantity, for simplicity of exposition. If an agent chooses to live on his human capital endowment and provide entertainment services, then his human capital affects the quality of the entertainment services in two ways. First of all, his human capital is unique in some dimensions, and therefore so are the services that he provides (for instance, consider the difference between Jay-Z and Madonna). As a result, each entertainer provides a unique variety of entertainment services, indexed by her identity  $i \in [0, 1]$  and different agents' entertainment services compete under monopolistic competition. Secondly, the entertainment service provided with a higher level of human capital is of a better quality, in the sense that it gives consumers a higher value, as will be shown when we come to the utility of the agents. Regarding the quantity of output, since we abstract from the effect of human capital on it, it follows that all agents have the same production function. Specifically, if an agent hires  $L$  units of labor, the output of her variety is

$$y = \begin{cases} \frac{A}{c}L, & \text{if } L \leq \frac{c}{A}B \\ B, & \text{if } L > \frac{c}{A}B \end{cases}, \quad (1)$$

Thus, if an agent decides to use his time in order to supply human capital rather than labor, thereby providing a variety of entertainment services, he can subsequently hire labor to produce output at constant returns to scale up to the limit  $B$ . Thus, the production of entertainment services displays Increasing Returns to Scale up to some limit (IRSL).

An alternative way to see this is to consider the cost function. Let  $F$  denote the opportunity cost of the agent's time<sup>10</sup> and  $w$  the wage of labor. Then the cost function associated with producing entertainment services is

$$C(y) = \begin{cases} F + w\frac{c}{A}y, & \text{if } y \leq B \\ \infty, & \text{if } y > B. \end{cases}, \quad (2)$$

This  $B$  marks the maximum scale of operation for an entertainer.

Agents have identical preferences. If an agent consumes  $s$  units of the subsistence good and  $e_i$  units of variety  $i$  of entertainment services, where  $i \in E$  and  $E$  is the set of varieties of entertainment

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<sup>10</sup>Since the alternative use of his time is to supply labor, we have that  $F = w$ .

services available on the market, then her utility is

$$\left( \mu s^{\widehat{\rho}} + \left( \int_E (h_i e_i)^\rho \right)^{\widehat{\rho}/\rho} \right)^{1/\widehat{\rho}},$$

where  $\mu > 0$  measures the relative importance of the subsistence good in the agent's utility function;  $\widehat{\rho} \in [0, 1)$  measures the substitutability between the subsistence good and entertainment services; and  $\rho \in (0, 1)$  measures the substitutability between one entertainment service and another. Assume  $\widehat{\rho} < \rho$ , namely that the subsistence good is less substitutable for entertainment services than one variety of entertainment service is to another. Note that the marginal value of agent  $i$ 's services is  $h_i$ , the same as the amount of her human capital. That is, the entertainment services provided with higher human capital deliver a greater value to consumers, as noted above.

We set the subsistence good as the numeraire. Let  $p_i$  denote the price of variety  $i$  of entertainment services and let  $m$  denote the income of an agent. Then, the consumption decision that the agent faces is

$$\begin{aligned} \max_{s, \{e_i\}_{i \in E}} & \left( \mu s^{\widehat{\rho}} + \left( \int_E (h_i e_i)^\rho \right)^{\widehat{\rho}/\rho} \right)^{1/\widehat{\rho}}, \\ \text{s.t.} & s + \int_E p_i e_i \leq m. \end{aligned}$$

His demand for the subsistence good and entertainment services are, respectively:<sup>11</sup>

$$\begin{aligned} s &= m \cdot \frac{1}{1 + \mu^{1/(\widehat{\rho}-1)} P^{\widehat{\rho}/(\widehat{\rho}-1)}} \\ e_i &= m \cdot f(P, \mu) \cdot h_i^{\rho/(1-\rho)} p_i^{-1/(1-\rho)}, \end{aligned} \tag{3}$$

where  $P$  is the general price of entertainment services and

$$P := \left( \int_E (p_i/h_i)^\rho \right)^{(\rho-1)/\rho}, \tag{4}$$

and the function  $f(P, \mu)$  is given by

$$f(P, \mu) := \frac{P^{\frac{\rho-\widehat{\rho}}{(1-\rho)(1-\widehat{\rho})}}}{\mu^{\frac{1}{1-\widehat{\rho}}} + P^{\frac{\widehat{\rho}}{\rho-1}}}.$$

Given price  $p$ , the aggregate demand for a particular variety of entertainment services of quality  $h$

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<sup>11</sup>In the model each professional sells to all agents and each agent buys from all professionals. This is a result of the fact that in the model consumers are homogenous, with identical utility functions. In reality, no musician sells to the entire population (with the possible exception of Michael Jackson in 1982) and no one buys music from all musicians. But if we aggregate the consumption of all music and imagine that it is consumed by one "representative agent", then the model makes sense in terms of tracking aggregate demand for each musician.

(also equal to the human capital endowment of the entertainer) is

$$D(p; h) = M \cdot f(P, \mu) \cdot h^{\rho/(1-\rho)} p^{-1/(1-\rho)}, \quad (5)$$

where

$$M := \int_{[0,1]} m_i \quad (6)$$

is aggregate income. Note that  $D'_h > 0$ . This is due to the fact that consumers derive greater value from a variety of service that is provided with relatively higher human capital, and hence have a larger demand for it, conditional on price.

If an agent with human capital  $h$  chooses to supply labor and produce the subsistence good, he gets  $A$ . Hence in equilibrium the wage of labor employed in the production of entertainment services is also  $A$ , that is,  $w = A$ . Therefore, by (2), the marginal cost of producing entertainment services up to scale  $B$  is  $w \frac{c}{A} = c$ , which is independent of  $A$ . If the agent chooses to live on his human capital and produce his variety of services, the demand for his services will be given by (5), where he takes the aggregate variables  $P$  and  $M$  as given. He then sets the price of his services by solving the following decision problem:

$$m(h) = \max_p (p - c)D(p; h), \text{ s.t. } D(p; h) \leq B \quad (7)$$

The agent chooses to provide entertainment services instead of supplying labor only if

$$m(h) \geq A \quad (8)$$

From the envelope theorem and (7),  $m'(h) = D'_h > 0$ . There thus exists a critical value  $k \in [0, 1]$  such that agent  $i$  chooses to provide entertainment services, if and only if  $i \geq k$ , where  $k$  is pinned down by

$$m(h_k) = A. \quad (9)$$

For  $i < k$ , agent  $i$  earns wage  $w = A$ , and for  $i \geq k$  agent  $i$  earns  $m(h_i)$ , the rents associated with her human capital. Hence the set of available entertainment services is  $E = [k, 1]$ . It follows that

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<sup>12</sup>More generally,  $k$  satisfies  $\left\{ \begin{array}{l} k = 0 \text{ if } m(h_0) > A \\ k = 1 \text{ if } m(h_1) < A \\ m(h_k) = A \text{ if } m(h_0) < A < m(h_1) \end{array} \right\}$ . The first two cases capture the possibilities

that no one produces the subsistence good and that no one produces any entertainment services. With CES preferences, neither occurs in equilibrium because if no one produces the subsistence good, the marginal utility from consumption of it will be infinitely large, and providing it will be very profitable. This argument also applies to the case in which no one provides entertainment services.

the general price for entertainment services, from (4), is given by

$$P = \left( \int_k^1 (p_i/h_i)^{\rho/(\rho-1)} \right)^{(\rho-1)/\rho}, \quad (10)$$

and the aggregate income is

$$M = kA + \int_k^1 m(h_i). \quad (11)$$

**Definition 1.** A profile  $(P, k, M)$  forms a competitive equilibrium if

- (i)  $P$  is given by (10), where  $p_i$  solves (7) with  $h = h_i$ ;
- (ii) agent  $i$  chooses to supply labor if and only if  $i < k$  where  $k$  is determined by (9);
- (iii) Aggregate income is given by (11).<sup>13</sup>

For technical reasons which we will explain, we assume that

$$\max_{x \in [k_0, 1]} \frac{h'_x/h_x}{1 + h'_x/h_x \cdot (x - k_0)} < \frac{1}{1 - k_0}, \quad (12)$$

where

$$k_0 = \frac{Bc}{A + Bc}.$$

This condition concerns the distribution of human capital and follows from a more intuitive condition, namely that  $[\log h(x)]' < 1/(1 - x)$ ,<sup>14</sup> which says that  $\log h(x)$  does not grow too fast.

#### 4. Equilibrium and Technological Change

In this section we prove the existence and uniqueness of an equilibrium, and then consider comparative statics with respect to  $B$ . In a final section we also consider a heretofore overlooked effect arising from an increase in  $A$ . We first focus on the case in which the capacity constraint,  $D(p; h) \leq B$ , is binding for any agents who choose to be entertainers, such that their profit is  $(p - c)B$ . This effectively requires  $B$  to be sufficiently small and an exact condition is provided in Subsection 4.3. In that subsection, we also show that the insights derived from this case can then be applied straightforwardly to the case in which the capacity constraint is not binding for some portion of the entertainers. Of course, if it is not binding for any agents then an increase in  $B$  will have no effect.

<sup>13</sup>We skip the clearing of the subsistence good market, which pins down the fraction of labor used for producing the good, a variable that is not very interesting in the context of this paper.

<sup>14</sup>This comes from  $\frac{h'_x/h_x}{1 + h'_x/h_x \cdot (x - k_0)} = [1/(h'_x/h_x) + x - k_0]^{-1} < [1 - x + x - k_0]^{-1} = \frac{1}{1 - k_0}$ .

Applying the binding capacity constraint,  $D(p, h_i) = B$ , the price of the variety of entertainment services provided by agent  $i$  is:

$$p_i = \left( \frac{Mf(P, \mu)}{B} \right)^{1-\rho} h_i^\rho. \quad (13)$$

Thus, an agent with higher human capital charges a higher price for her services because they deliver a higher value to consumers. In fact, the price is proportional to the marginal value raised to power  $\rho < 1$  – that is,  $h_i^\rho$  – because one variety of entertainment services is not a perfect substitute for another, in general. In the special case in which it is – that is  $\rho = 1$  – the price of a variety is then directly proportional to its marginal value.

With the price of each variety given by (13), the price index, from (10), is

$$P = \left( \frac{Mf(P, \mu)}{B} \right)^{1-\rho} H_k^{\rho-1}, \quad (14)$$

where

$$H_k := \left\{ \int_k^1 h_i^\rho \right\}^{\frac{1}{\rho}}. \quad (15)$$

This equation for the general price of entertainment services leads to two results.

One, it implies that  $M/(BH_k) = P^{\frac{1}{1-\rho}}/f(P, \mu)$ . With  $f(P, \mu) = \frac{P^{\frac{\rho-\hat{\rho}}{(1-\rho)(1-\hat{\rho})}}}{\mu^{\frac{1}{1-\hat{\rho}}} + P^{\frac{\hat{\rho}}{\rho-1}}}$ , it follows that

$$BH_k P + BH_k (\mu P)^{\frac{1}{1-\hat{\rho}}} = M, \quad (16)$$

where  $BH_k P$  is the aggregate spending on entertainment services. To see this, note that from (3) the fraction of each agent's income, and hence of aggregate income, spent on the subsistence good is  $\frac{1}{1+\mu^{\frac{1}{1-\hat{\rho}}}} P^{\frac{\hat{\rho}}{\rho-1}}$ . Thus, the fraction spent on entertainment services is  $1 - \frac{1}{1+\mu^{\frac{1}{1-\hat{\rho}}}} P^{\frac{\hat{\rho}}{\rho-1}} = \frac{1}{1+\mu^{\frac{1}{1-\hat{\rho}}}} P^{\frac{\hat{\rho}}{\rho-1}}$ . Note that for the Cobb-Douglas case, where  $\hat{\rho} = 0$ , this fraction is  $1/(1+\mu)$ , and thus is independent of the price of entertainment,  $P$ . Moreover, from equation (16)  $BPH_k = \left( \frac{1}{1+\mu^{\frac{1}{1-\hat{\rho}}}} P^{\frac{\hat{\rho}}{\rho-1}} \right) M$ . Hence,  $BPH_k$  is indeed the aggregate spending on entertainment services. Given that this spending and the aggregate spending on the subsistence good add up to aggregate income  $M$ , from equation (16)  $BH_k (\mu P)^{\frac{1}{1-\hat{\rho}}}$  is the aggregate spending on the subsistence good.

The other result from (14) is  $\left( \frac{Mf(P, \mu)}{B} \right)^{1-\rho} = PH_k^{1-\rho}$ . Substituting this into (13), we have  $p_i = PH_k^{1-\rho} h_i^\rho$ . Since the profit of an entertainer  $i$  is  $m(h_i) = (p_i - c)B$ , then

$$m(h_i) = BPH_k^{1-\rho} h_i^\rho - Bc, \quad (17)$$

The first term on the right hand side reflects revenue, which we denote by  $R(h_i)$ , while the second

term reflects labor costs. The revenue is proportional to capacity, the general price of entertainment services, and the entertainer's human capital raised to the power  $\rho$ . The first two factors are common to all the entertainers. Hence, an entertainer's revenue is proportional to his human capital raised to the power  $\rho$  – i.e., the price she charges. Indeed the fraction of aggregate spending on entertainment services that flows to entertainer  $i$  is  $h_i^\rho / \int_k^1 h_j^\rho = h_i^\rho / H_k^\rho$  because  $R(h_i) = BPH_k \times h_i^\rho / H_k^\rho$  and we have seen the aggregate spending on entertainment services is  $BPH_k$ .

Having found the profit of each entertainer, we now move on to determining the occupational choice threshold,  $k$ , in two steps. First, agent  $k$ , who is indifferent between becoming an entertainer or supplying labor, is identified by the condition  $m(h_k) = A$ . Applying (17), for  $i = k$  it follows that

$$PH_k^{1-\rho} h_k^\rho = c + \frac{A}{B}. \quad (18)$$

Note that the term on the right hand side is the price charged by the marginal entertainer, i.e. agent  $k$ . As the marginal entertainer, her profit is  $A$  and hence the price she charges satisfies  $p_k \times B - Bc = A$ . Hence

$$p_k = A/B + c.$$

Second, with the profit of individual entertainers given by (17), aggregate income, from (11), is:

$$M = kA - (1 - k)cB + BPH_k. \quad (19)$$

As noted above, the last term,  $BPH_k$ , is the aggregate spending on entertainment services. The first two terms represent the aggregate supply of the subsistence good (which, in equilibrium, equals the aggregate spending on the good). Specifically, agents  $j \in [0, k]$  are supplying labor to the economy, of whom (from 1)  $\frac{c}{A}B \times (1 - k)$  agents are working for entertainers. Thus,  $k - \frac{c}{A}B \times (1 - k)$  agents work to produce the subsistence good, yielding an output of  $[k - \frac{c}{A}B \times (1 - k)] \times A = kA - (1 - k)cB$ . Note that this aggregate supply of the subsistence good can be re-written as  $(A + Bc)(k - k_0)$ , where  $k_0 = \frac{Bc}{A+Bc}$  is the threshold for the number of agents at which the aggregate supply of the subsistence good is zero.

Canceling out  $M$  using (16) and (19), we have

$$BH_k(\mu P)^{\frac{1}{1-\rho}} = (A + Bc)(k - k_0).$$

As we have seen above, the left hand side of this equation is aggregate spending on, and the right hand side is the aggregate supply of, the subsistence good. As a result, this equation reflects market clearing for the subsistence good. Substituting for  $P$  with the solution from (18) and re-arranging,

we arrive at a single equation that pins down  $k$  in equilibrium:

$$\mu^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}} \times (A/B + c)^{\frac{\hat{\rho}}{1-\hat{\rho}}} = k - k_0 \quad (20)$$

Again, as noted above, the term on the right hand side of this equation is the aggregate supply of the subsistence good, measured in units of the revenue of the marginal entertainer (i.e.,  $A + Bc$ ), if his identity is  $k$ . The term on the left hand side of this equation, as we have explained above, is the aggregate spending on the subsistence good conditional on the marginal entertainer being agent  $k$ , also measured in units of his revenue. This can be clearly seen for the Cobb-Douglas case, where  $\hat{\rho} = 0$ . In this case, this term simplifies as  $\mu H_k^\rho / h_k^\rho$ . Measured in units of the agent's revenue, the spending on his service (the agent's revenue) is 1. Since this spending is  $h_k^\rho / H_k^\rho$  of the aggregate spending on entertainment services, the aggregate spending on entertainment services is therefore  $H_k^\rho / h_k^\rho$ , and then  $\mu$  times this term gives the aggregate spending on the subsistence good in the Cobb-Douglas case, where the ratio of the spending on the subsistence good to that on entertainment services is always  $\mu$ , independent of the price index,  $P$ .<sup>15</sup> For the non-Cobb-Douglas case, this ratio depends on the price and hence we have the additional term  $(A/B + c)^{\frac{\hat{\rho}}{1-\hat{\rho}}} = (p_k)^{\frac{\hat{\rho}}{1-\hat{\rho}}}$ . In particular, if entertainment services are cheaper, reflected in a smaller  $p_k$ , then spending on the subsistence good is reduced, as it becomes relatively more expensive (in the case of  $\hat{\rho} > 0$  which means that the price effect dominates the income effect).

The aggregate supply of the subsistence good – the right hand side term – increases with  $k$  due to the fact that the larger is  $k$ , the more agents there are that will supply labor – i.e., fewer agents will work for entertainers and will instead produce the subsistence good. At the same time, aggregate spending – on the left hand side – decreases with  $k$ , the identity of the marginal entertainer, and spending goes to zero as  $k$  goes to 1.<sup>16</sup> Intuitively, this is because an agent with relatively little human capital chooses to become an entertainer only if the economy is rich enough such that a large enough aggregate income is spent on entertainment services. Put differently, if the marginal entertainer has a relatively high human capital level – i.e.,  $k$  is big – then the economy must be poorer, which means the aggregate spending on the subsistence good is smaller too. In the extreme case, if the economy can support only the agent with the greatest human capital as an entertainer – i.e.,  $k = 1$  – then it must be extremely poor in that aggregate income approaches zero and each agent can only spare a tiny amount of income to spend on entertainment services.

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<sup>15</sup>In the Cobb-Douglas case, each agent spends a fraction of  $\frac{\mu}{1+\mu}$  of his income on the subsistence good and that of  $\frac{1}{1+\mu}$  on entertainment services. Hence the aggregate spending on the former good is  $\mu$  times that on the latter good.

<sup>16</sup>Since  $\rho - \hat{\rho} > 0$ ,  $H_k^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}}$  increases with  $H_k = \{\int_k^1 h_i^\rho\}^{\frac{1}{\rho}}$ , which decreases with  $k$ . Since  $\rho > 0$ ,  $h_k^{\frac{-\rho}{1-\hat{\rho}}}$  decreases with  $h_k$  which, by assumption, increases with  $k$ . Moreover,  $H_1 = 0$ . Hence the term on the left hand side equals 0 at  $k = 1$ .

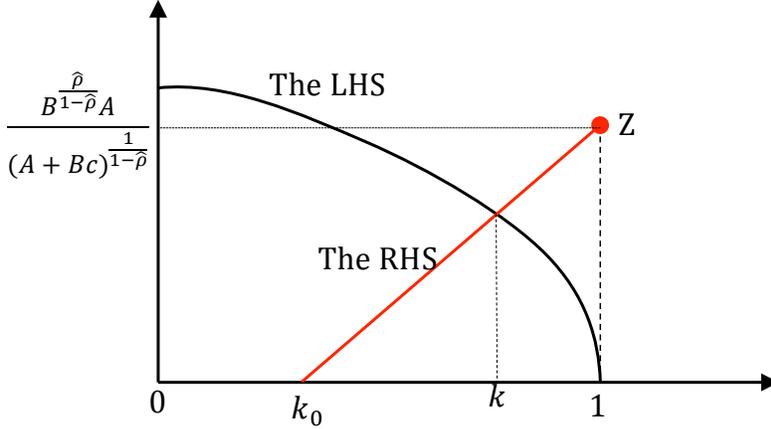


Figure 2: The Existence and Uniqueness of Equilibrium

Equation (20) can be re-arranged into

$$\mu^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\hat{\rho}-\rho}{1-\hat{\rho}}} h_k^{-\frac{\rho}{1-\hat{\rho}}} = (A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}} (k - k_0), \quad (21)$$

As argued above, the left hand side of (21) decreases from a positive number to 0 with  $k$  ascending from  $k_0$  to 1, and with this movement of  $k$  the right hand side of (21) linearly increases from 0 to some positive number. Both sides are depicted in Figure 2. This argument indicates leads to the following proposition:

**Proposition 1.** *A unique equilibrium exists, in which  $k \in (k_0, 1)$  and is given by (21).*

The economic intuition for the existence and uniqueness of an equilibrium can be understood in light of the CES preferences and the market forces at play. The former ensures that both the subsistence good and some entertainment services are provided in any equilibrium, such that the population is divided between these two occupation types – i.e.,  $k$  lies between 0 and 1. Market forces then ensure the uniqueness of this division: if too many agents choose to provide labor and produce the subsistence good, then the entertainment services will be costly, which will induce entry into entertainment service provision. Conversely, if too few agents provide labor there will be entry into production of the subsistence good.

Next consider the equilibrium income distribution. Agents  $i < k$  choose to provide labor and earn income  $A$ , while agents  $i \geq k$  become entertainers, in which case their income,  $m_i$ , is related to their human capital according to (17). Simplifying this equation with  $PH_k^{1-\rho} = (c + A/B)h_k^{-\rho}$  (from equation (18)) we find that the equilibrium income distribution is:

$$m_i = \left\{ \begin{array}{l} A \text{ if } i < k \\ (Bc + A) \frac{h_i^p}{h_k^p} - Bc \text{ if } i \geq k \end{array} \right\} \quad (22)$$

This income distribution is illustrated in Figure 3.<sup>17</sup>

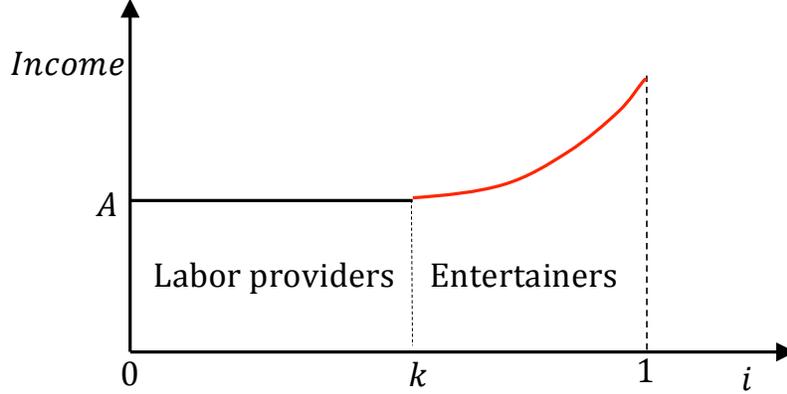


Figure 3: The Equilibrium Income Distribution

#### 4.1. New Technologies that Expand the Limit of IRS

Here we consider the comparative statics with respect to  $B$ , the IRS limit.<sup>18</sup> We consider first how an increase in  $B$  affects the occupational choice of the agents, captured by  $k$ , and then consider its effect on the income distribution. The equilibrium  $k$  is determined by equation (20), which is based on the market clearing of the subsistence good. From this equation, we can see that an increase in  $B$  generates two effects that impact  $k$ . First, the term on the right hand side of (20), which represents the aggregate supply of the subsistence good, goes down because  $k_0 = Bc/(A + Bc)$  increases with  $B$ . Intuitively, a larger  $B$  means that more labor is required as input into the production of entertainment services. Hence, given the total supply of labor (i.e.,  $k$ ), fewer workers are employed to produce the subsistence good and the less of the good is produced. As result, given the demand for the subsistence good, a rise in  $B$  implies that more agents will supply labor in order to meet the demand. That is, the supply-side effect alone drives  $k$  up. The second effect, from the left hand side of (20) (which represents the aggregate demand for the subsistence good), is that the

<sup>17</sup>The figure is based on the assumption that  $h_i$  is a convex function of  $i$  so that  $m_i$ , though a concave function of  $h_i$ , is convex in  $i$ . Roughly, the assumption is that within a typical talent distribution, there are a small number of people at the top who are much more talented than the rest – a view that seems consistent with the evidence.

<sup>18</sup>Since we are examining the case in which the capacity constraint,  $D(p; h) \leq B$ , is binding, the comparative statics are based on the assumption that it remains binding following the change we are considering. Later we consider the comparative statics for the case in which the capacity constraint is binding for some share of entertainers.

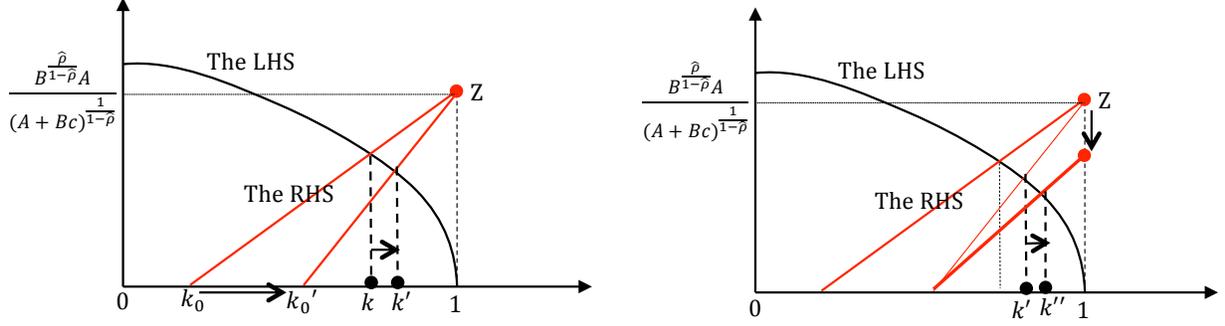


Figure 4: The effect of an increase in  $B$  on  $k$ . The left panel: an increase in  $B$  moves  $k_0$  to  $k_0'$ , which increases  $k$  to  $k'$ . The right panel: if point  $Z$  moves down, then  $k$  shifts further to  $k''$

price adjustment term,  $(A/B + c)^{\frac{\hat{\rho}}{1-\hat{\rho}}}$ , decreases with  $B$ . Intuitively, when the quantity of each variety increases, entertainment services in general become relatively cheaper, which reduces the demand for the subsistence good when  $\hat{\rho} > 0$ . This demand-side effect alone reduces demand for the subsistence good and hence decreases the number of agents that provide labor; that is, it drives  $k$  down.

These two effects therefore drive  $k$  in opposite directions. The trade-off between them determines its ultimate direction – up or down. As we saw, the demand side effect depends on the value of  $\hat{\rho}$  and vanishes at  $\hat{\rho} = 0$  where the price effect is completely offset by the income effect. By continuity, we expect this effect to be weak and dominated by the positive effect on the supply side – hence  $k$  to go up – if  $\hat{\rho}$  is close enough to zero. To find a sufficient condition for this dominance, we turn to equation (21), the two sides of which are pictured in Figure 2. We note that the left hand side is independent of  $B$ . As a result, the curve in Figure 2, representing the LHS, is invariant to an increase in  $B$ . The RHS, on the other hand, is affected in two ways. First,  $k_0 = Bc/(A + Bc)$  increases with  $B$  so that  $k_0$  moves rightward, driving  $k$  to rise to  $k'$ . Second, the uppermost part of the line,  $Z$ , may shift up or down. If the position of  $Z$  does not change, while  $k_0$  moves to the right, clearly  $k$  will also move to the right (to the position of  $k'$ ), as is illustrated in the left panel of Figure 4. If  $Z$  moves down then  $k$  shifts further to the right (to the position of  $k''$ ), as is illustrated in the right panel of the Figure.

The height of  $Z$  is  $(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}(1 - k_0) = AB^{\frac{\hat{\rho}}{1-\hat{\rho}}}/(A + Bc)^{\frac{1}{1-\hat{\rho}}}$ . Thus,  $Z$  moves down with an increase in  $B$  if

$$\frac{dAB^{\frac{\hat{\rho}}{1-\hat{\rho}}}/(A + Bc)^{\frac{1}{1-\hat{\rho}}}}{dB} \leq 0,$$

which is equivalent to

$$c \geq \frac{\hat{\rho}}{1 - \hat{\rho}} \cdot \frac{A}{B} \quad (23)$$

It follows that

**Proposition 2.** *If  $c \geq \frac{\hat{p}}{1-\hat{\rho}} \cdot \frac{A}{B}$ , then  $dk/dB > 0$ . That is, with an increase in the limit of IRS, fewer agents choose to provide entertainment services, and the number of varieties provided falls.*

*Proof.* We relegate the proof to Appendix A.1. ■

Note that in this case agent  $i$ 's net gain from becoming a professional relative to providing labor is  $(p_i - c) \times B - A$ . If an increase in  $B$  pushes the agent out of the profession – that is, if it makes this net gain become negative – then this must be due to a decrease in  $p_i$ , that is, an effect that the price of his variety has to go down because of intensified competition. This is a difference to Melitz (2003), where the price of each variety does not respond to changes in the number or prices of competing varieties.

According to the proposition, if  $\hat{\rho}$  is small – so the price effect is mostly cancelled by the income effect – then the marginal entertainer is competed out of the entertainment occupation to become labor providers as a result of an increase in  $B$ .

Having examined the effect of an increase in  $B$  on occupation choice, we move on to considering its effects on the income distribution. Two effects are presented below. The first is a direct implication of Proposition 2: fiercer competition due to an increase in  $B$  generates losses for lower-end entertainers. Consider those entertainers endowed with a level of human capital close to the marginal entertainer's, and who are therefore squeezed out of the entertainment business with the increase in  $B$ . Before the rise in  $B$  they earned strictly more than the wage of labor,  $A$ , as they strictly preferred being an entertainer to providing labor. After the increase in  $B$  they are squeezed out and subsequently provide labor, and therefore earn the wage of labor. These agents therefore lose. This result is stated as the proposition below and is formally proved.

**Proposition 3.** *There exists  $\hat{k} > k$  such that  $dm_i/dB < 0$  for  $i \in (k, \hat{k})$  – namely, the lower-end entertainers lose from an increase in the limit of IRS.*

*Proof.* We relegate the proof to Appendix A.2. ■

The second effect of a rise in  $B$  for the income distribution is that it increases income equality within the entertainment occupation, as the following proposition shows.

**Proposition 4.** *Under condition (12), for  $i > k$ ,  $dm_i/dB$  increases with  $i$  and hence is positively correlated with  $m_i$ .*

*Proof.* We relegate the proof to Appendix A.3. ■

That is, the higher the current income of an entertainer, the more the entertainer gains (or the less he loses) from an increase in the limit of IRS, which leads to growth in income inequality within the entertainment occupation. The intuition for the proposition is as follows. An increase in  $B$  generates three effects on the revenues of entertainers, aside from increasing their cost,  $Bc$ . First, a positive effect: a rise in  $B$  enlarges entertainers' capacity and thereby increases their revenues. Second, a negative effect: since all entertainers are equally exposed to the increased capacity, the competition between them becomes fiercer, resulting in a lower general price of entertainment services, which reduces revenues (all else equal). And third, an increase in  $B$  may affect aggregate income, thereby affecting entertainers' revenues. The sign of this effect is unclear, *a priori*. Setting the third effect aside – more discussion regarding this effect is presented below – all entertainers face fiercer competition to the same degree, since they face the same lower general price of entertainment services, but an entertainer who has relatively more human capital – and thus earns relatively more – receives a greater gain from the enlargement in capacity because he provides a better quality of service and is therefore able to charge a higher price for his variety.<sup>19</sup> As a result, entertainers with initially higher earnings gain more or lose less from an increase in  $B$ . Indeed, Proposition 3 shows that if an entertainer's human capital is low enough, then his gains from the enlargement of capacity is dominated by the losses due to fiercer competition and raised labor costs.

With an increase in the limit of IRS for any particular occupation the third effect above, which operates via the impact on aggregate income, should not be salient due to the fact that, in reality, economies consist of hundreds of occupations, such that any change in one occupation is unlikely to have a large effect on aggregate income. However, in the model economy there is only one human-capital-intensive occupation – namely entertainment – and hence an increase in the limit of IRS for this occupation does in fact generate a salient effect on the aggregate economy. Indeed, if the condition assumed in (12) does not hold, this effect can be so negative that Proposition 4 is invalidated and  $dm_i/dB$  decreases with  $m_i$ . In Appendix B, we construct an example of this case. Intuitively, if due to a decrease in aggregate income, the aggregate spending on entertainment services is reduced by  $\Delta M$  then, other things fixed, the revenue of entertainer  $i$  is reduced by  $h_i^\rho/H_k^\rho \times \Delta M$  due to the fact that she acquires a fraction  $h_i^\rho/H_k^\rho$  of that spending. Hence, if aggregate income falls, entertainers who currently earn relatively more suffer a relatively greater loss – namely,  $dm_i/dB$  is negatively correlated with  $m_i$ .

As noted previously, the gains to an entertainer from the enlargement of capacity is in proportion to her human capital endowment raised to the power  $\rho$  (as is the price she charges). If an entertainer's human capital endowment is high enough the increase in revenue will outweigh the losses due to fiercer competition and higher labor costs, and the entertainer will reap a net gain due

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<sup>19</sup>The price he charges, by (13), is in proportion to  $h_i^\rho$ .

to the increase in  $B$ . To state this formally, let

$$\Omega(\rho) := \max_{k \in [k_0, 1]} \frac{\rho \cdot h'_k / h_k}{1 + \rho \cdot h'_k / h_k \cdot (k - k_0)}.$$

By assumption (12),  $\Omega(\rho) \cdot A / (A + Bc) < 1$ .<sup>20</sup> We can then state the following:

**Lemma 1.**  *$dm_i/dB > 0$ , namely agent  $i$ 's income rises with an increase in the limit of IRS, if*

$$\frac{h_i}{h_k} > \left( \frac{1}{1 - \Omega(\rho) \cdot A / (A + Bc)} \right)^{\frac{1}{\rho}}. \quad (24)$$

*Proof.* We relegate the proof to Appendix A.4. ■

Condition (24), however, is not easy to check. This is because  $k$  is determined in equilibrium and depends on the distribution of human capital (specifically, the functional form of  $h(i)$ ). Below, we present an approach, dispensing with  $k$ , to get a condition under which the top entertainers gain on net from an increase in the limit of IRS.

Let  $f(k_0, y)$  denote the unique solution for  $t \in [k_0, 1]$  in

$$t - k_0 = y(1 - t)^{\frac{\rho - \hat{\rho}}{\rho(1 - \hat{\rho})}},$$

and let

$$D := \mu^{\frac{1}{1 - \hat{\rho}}} (A/B + c)^{\frac{\hat{\rho}}{1 - \hat{\rho}}}.$$

**Lemma 2.** *Assume  $h_1 > 1$ . If for some  $\zeta$ ,  $h_1 \geq \zeta \cdot h(f(k_0, D \cdot \zeta^{\frac{\rho}{1 - \hat{\rho}}}))$ , then  $h_1 > \zeta \cdot h_k$ .*

*Proof.* We relegate the proof to Appendix A.5. ■

The two lemmas above lead to the following proposition, which gives a condition for the distribution function of human capital under which the top entertainers' income strictly increases with  $B$ . Let

$$\xi := \left[ \frac{1}{1 - \Omega(\rho) \cdot A / (A + Bc)} \right]^{\frac{1}{\rho}}.$$

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<sup>20</sup>By the envelope theorem,  $\Omega(\rho)$  increases with  $\rho$ . Therefore,  $\Omega(\rho) \leq \Omega(1) = \frac{h'_k / h_k}{1 + h'_k / h_k \cdot (k - k_0)}$ , which by the assumption is smaller than  $\frac{A + Bc}{A}$ .

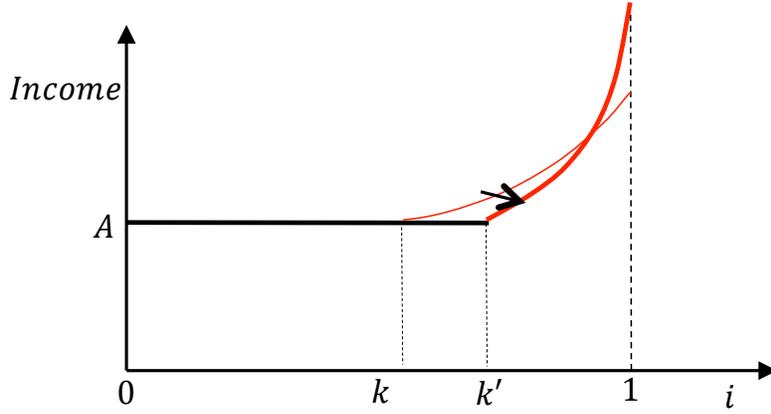


Figure 5: An increase in  $B$  squeezes the lower-end entertainers out, and raises income inequality within the entertainment occupation.

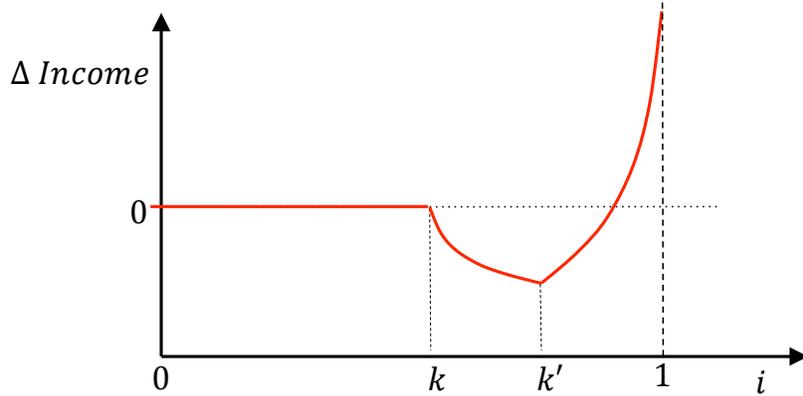


Figure 6: Income growth due to an expansion in  $B$ .

**Proposition 5.** *If  $h_1 > 1$  and  $h_1 \geq \xi \cdot h(f(k_0, D \cdot \xi^{\frac{\rho}{1-\rho}}))$ , then  $dm_1/dB > 0$ .*

When this proposition holds, the top entertainers gain on net from an increase in the limit of IRS. This result, together with Proposition 2, which states that entertainers at the bottom of the distribution are pushed out of the entertainment occupation into providing unskilled labor, implies that an increase in  $B$  leads to the change in the income distribution depicted in Figure 5. The resulting pattern of income changes across agents is illustrated in Figure 6.

#### 4.2. An Increase in the Productivity of Unskilled Labor

We now consider the comparative statics with respect to  $A$ , the productivity of (unskilled) labor. As in the case of an expansion in  $B$ , we first explore how an increase in  $A$  affects agents' occupational

choices and, second, we explore its effect on the income distribution. To begin, observe that a rise in  $A$  directly increases the income of labor, but has no direct impact on the income of entertainers. That is because although an increase in labor productivity increases the productivity of entertainment services since fewer workers are needed to produce the same amount of entertainment, it meanwhile increases the wage of labor, and on net, the marginal (labor) cost of producing entertainment services,  $w \times c/A$ , stays constant at  $c$  from (2). In this sense we say that an increase in  $A$  is biased toward (unskilled) labor. It seems at first glance that an increase in  $A$  would induce more agents to provide labor, fewer to become entertainers, and would reduce income inequality. However, we show below that these direct effects are in fact fully offset by a general equilibrium effect. Specifically, an increase in  $A$  raises aggregate income, which will raise the spending on entertainment services and consequently enrich entertainers. This general equilibrium effect, we will show, dominates the direct effect for the impact on both occupation choice and the income distribution.

To consider the impact on occupation choice, we return to equation (20), which determines equilibrium  $k$  via market clearing of the subsistence good. The supply side (the right hand side of the equation) increases with  $A$  since  $k_0 = Bc/(A + Bc)$  decreases with it. Intuitively, given the quantity of labor  $k$ , output of the subsistence good rises with labor productivity. For fixed demand, this effect on the supply side induces fewer agents to produce the subsistence good – that is it induces  $k$  to go down. On the demand side (the left hand side of equation 20), the price adjustment factor,  $(A/B + c)^{\frac{\hat{\rho}}{1-\hat{\rho}}}$ , also increases with  $A$  as long as if  $\hat{\rho} > 0$ . Intuitively, a larger  $A$ , by inducing a greater supply of the subsistence good, makes the entertainment services relatively more expensive and increases the demand for the subsistence good in the non-Cobb-Douglas case. This effect on the demand side induces  $k$  to go up. The effect, however, vanishes if  $\hat{\rho} = 0$  and as a consequence we expect that it will be dominated by the negative effect on the supply side when  $\hat{\rho}$  is small. To find a sufficient condition for this dominance, we again go to equation (21), the two sides of which are depicted in Figure 7. The LHS, represented by the curve, is independent of  $A$ . Thus, the curve in Figure 7 does not shift with an increase in  $A$ . As for the RHS, an increase in  $A$  shifts the straight line in Figure 7 in two ways. First,  $k_0 = Bc/(A + Bc)$  falls with an increase in  $A$  and the position of  $k_0$  shifts leftward to the position of  $k'_0$ . Second, the uppermost part of the line,  $Z$ , may move up or down. If the position of  $Z$  does not change, but  $k_0$  moves leftward, then so does  $k$ , to  $k'$ , as is illustrated by the left panel of the Figure. If  $Z$  moves upward, then  $k$  falls further to  $k''$ , as is illustrated by the right panel of the Figure.

The height of  $Z$  is  $(A/B + c)^{\frac{\hat{\rho}}{1-\hat{\rho}}}(1 - k_0) = AB^{\frac{\hat{\rho}}{1-\hat{\rho}}}/(A + Bc)^{\frac{1}{1-\hat{\rho}}}$ .  $Z$  moves upward with an increase in  $A$  if

$$\frac{dAB^{\frac{\hat{\rho}}{1-\hat{\rho}}}/(A + Bc)^{\frac{1}{1-\hat{\rho}}}}{dA} \geq 0,$$

which is equivalent to (23). Therefore,

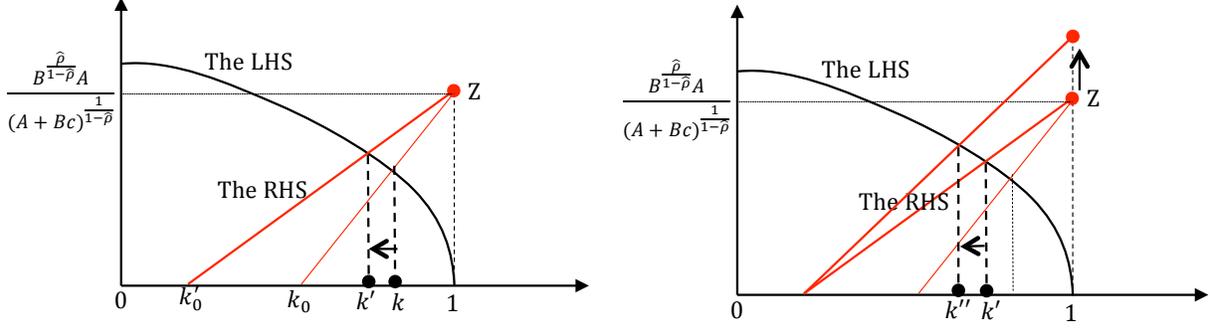


Figure 7: The effect of an increase in  $A$  on  $k$ . The left panel: an increase in  $A$  moves  $k_0$  to the left, which decreases  $k$  to  $k'$ . The right panel: if  $Z$  moves upward, then  $k$  falls further to  $k''$

**Proposition 6.** *If  $c \geq \frac{\hat{\rho}}{1-\hat{\rho}} \cdot A/B$  – i.e., (23) holds – then  $dk/dA < 0$ .*

*Proof.* We relegate the proof to Appendix A.6. ■

Thus, with a rise in labor productivity more agents choose to provide entertainment services, and the number of varieties therefore increases. This means that the general equilibrium effect dominates the direct effect in the impact of an increase in  $A$  on occupation choice.

Having examined the effect of an increase in  $A$  on occupation choice, we move on to considering its effects on the income distribution. We noted that an increase in  $A$  directly benefits unskilled labor, but has no direct impact on the income of entertainers. However, we have also noted that entertainers will gain indirectly from the general equilibrium effect. In fact, they gain more than the labor providers by the following proposition, and the more they currently earn, the more they gain. Hence, the general equilibrium effect dominates the direct effect here as well. To understand this proposition, note that if agent  $j$  provides labor, then his income is  $m_j = A$  and hence  $dm_j/dA = 1$ .

**Proposition 7.** *If  $i \geq k$ , namely if agent  $i$  is an entertainer, then  $\frac{dm_i}{dA} > 1$ . Moreover,  $\frac{dm_i}{dA}$  increases with  $i$  and hence is positively correlated with  $m_i$ .*

*Proof.* We relegate the proof to Appendix A.7. ■

Of the two results presented in the proposition, the second one can be intuitively explained as follows. The major effect of an increase in labor productivity is to raise aggregate income. With the economy becoming richer, the agents spend more on entertainment services. As an entertainer, agent  $i$  acquires a fraction  $h_i^p/H_k^p$  of the aggregate spending on entertainment services. Hence, the more an entertainer currently earns, i.e., the higher is her human capital, the greater is the growth in her income from an increase in  $A$ .

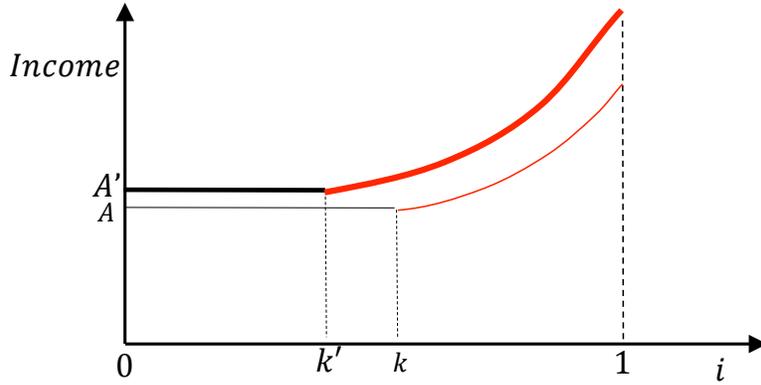


Figure 8: An increase in  $A$  raises all agents' incomes while also increasing income inequality.

To understand the first result – namely, that all the entertainers gain more than all labor providers due to an increase in  $A$  – we only need to understand why the marginal entertainer gains more than any of the labor providers. This is a direct implication of the change in occupation choice, as stated in Proposition 6. The marginal entertainer earns  $m_k = A$ . If her earnings rise with  $A$  less than one to one, she would strictly prefer to provide labor with the increase in  $A$  and then  $k$  would increase, which is not the case by Proposition 6. Hence, the marginal entertainer gains more than any labor provider due to an increase in  $A$ .

By Proposition 7, for the whole population, the more an agent currently earns, the more he gains from an increase in  $A$ . Therefore, *a rise in the productivity of unskilled labor increases overall income inequality*. The effect on the income distribution is illustrated in Figure 8.

### 4.3. Discussion

#### *When the Capacity Constraint Is Non-Binding for Some Entertainers*

Thus far we have considered the case in which the capacity constraint,  $D(p; h) \leq B$ , is binding for all entertainers. If the capacity constraint is non-binding for some entertainers, then these entertainers' human capital will lie at the lower end of the distribution. The demand for an entertainer's services, by (5), is proportional to  $h_i^{\rho/(1-\rho)}$ . Thus, the profit-maximizing output in the absence of the capacity constraint increases with  $h_i$ . As a result, if it is binding for agent  $i$  then it is binding for all the agents  $i' \geq i$ , and if it is not binding for agent  $i$ , then neither is it for any agent  $i' \leq i$ . Thus, if and only if the capacity constraint is binding for the marginal agent  $k$ , will it be binding for all entertainers. Since the entertainers' problem is given by (7), in the absence of a capacity constraint, the optimal price is  $c/\rho$ . The constraint is binding for agent  $k$  if he cannot reach this price by supplying enough output, namely if the price pinned down by the binding capacity constraint,  $p_k$ ,

is above  $c/\rho$ . This condition, with  $p_k$  given by (13) with  $i = k$ , formally is:

$$\left(\frac{Mf(P, \mu)}{B}\right)^{1-\rho} h_k^\rho > \frac{c}{\rho}. \quad (25)$$

Hence if this condition holds, then the relevant equilibrium is the case in which the capacity constraint is binding for all the entertainers, and the previous analysis holds.

If the condition does not hold, then the capacity constraint is binding for some share of entertainers and non-binding for the remainder. The argument above implies that there exists  $j \in (k, 1)$  such that it is non-binding for  $i < j$  and binding for  $i > j$ . In particular, it is non-binding for the marginal entertainer,  $k$ . In this case, the propositions derived above all hold true qualitatively.

Proposition 1 still holds. The unique equilibrium still exists, and is driven by the same economic forces as before. If too many agents choose to provide labor and produce the subsistence good, then the entertainment services will be expensive, which will induce further entry. Conversely, if too few agents provide labor there will be entry into production of the subsistence good.

Proposition 2 still holds, and therefore so does Proposition 3 which is driven by Proposition 2; that is, an increase in  $B$  squeezes the entertainers at the lower end out of the profession (i.e.,  $dk/dB > 0$ ). In fact, this holds even under a condition less strict than (23). That is because the marginal entertainer, now with a non-binding capacity constraint even with the present value of  $B$ , gains nothing from an increase in  $B$ , whereas in the case of his capacity constraint being binding, he obtains a positive effect due to the loosening of the constraint. In the absence of this positive effect, the marginal entertainer is pushed out of the profession by an even stronger forces.

Propositions 4 and 5 hold qualitatively, that is, the entertainers currently earning more will gain more or lose less from an increase in  $B$ , and if the top entertainer's human capital is high enough, then she gains on net from this increase. Both propositions are driven by the fact that entertainers with higher human capital – who therefore earn more – gain more from a capacity enlargement, again because they are able to charge higher prices as they provide higher valued services. But the exact conditions for these two propositions will change since  $M$ ,  $P$  and  $k$  will be ruled by a different profile of equilibrium conditions.

Proposition 6 holds qualitatively – namely, an increase in  $A$  induces more agents to seek work employing their human capital – though the exact condition may change. For the Cobb-Douglas case where  $\hat{\rho} = 0$  this is true and hence it is also true if  $\hat{\rho}$  is close enough to 0. It holds true for the Cobb-Douglas case due to the same intuition. To see this, let us go back to equation (20), which determines equilibrium  $k$  via market clearing of the subsistence good in units of the revenue of the marginal entertainer. Again, given  $k$ , his identity, an increase in  $A$  increases the supply of the good, while the aggregate demand is not changed in the the Cobb-Douglas case because both the proportion of aggregate income to the marginal entertainer's income and the fraction of the

aggregate income spent on the subsistence good are unchanged in this case. Therefore, an increase in  $A$  must reduce the number of agents who supply labor, that is, moves  $k$  leftward.

Proposition 7 still holds, namely relatively more talented (and thus richer) entertainers gain relatively more from an increase in  $A$ . Again it is driven by the same effect: an increase in  $A$  affects entertainers' income by raising aggregate income, and a bigger fraction of this increase accrues to an entertainer with higher human capital because he acquires a bigger fraction of the aggregate spending on entertainment services.

### *Unaffected Occupations*

The model thus far assumes that there is only one type of human capital, which is used to provide entertainment services. In reality, there are many types of human capital associated with many types of occupations. Moreover, as we argued in the Introduction, recent ICT innovations have led to a rise in the limit of IRS for some occupations, while for others – such as doctors or watch repairers – these innovations have had little impact on their scales of operation. This subsection examines how the increase in the limit of IRS for one occupation, which we refer to as the “affected” occupation, may impact another occupation, for which the limit of IRS is unchanged, which we refer to as the “unaffected” occupation.

Suppose that in addition to the continuum of agents previously described, there is now another continuum of agents,  $j \in [0, 1]$ . Agent  $j$  has one unit of labor and  $\tilde{h}_j$  of another type of human capital which is needed to produce another type of service (the unaffected service). Thus, each agent  $j$  makes an occupational choice between providing labor and providing services. The production of services is similarly subject to IRS up to limit  $\tilde{B}$  (this  $\tilde{B}$ , for example, denotes the maximum number of patients that a doctor can see):

$$y = \left\{ \begin{array}{l} \frac{A}{\tilde{c}}L \text{ if } L \leq \frac{\tilde{c}}{A}\tilde{B} \\ \tilde{B} \text{ if } L > \frac{\tilde{c}}{A}\tilde{B} \end{array} \right\}.$$

Each agents' utility is given by

$$\left( \mu s^{\hat{\rho}} + \left( \int_E (h_i e_i)^\rho \right)^{\hat{\rho}/\rho} + \left( \int_F (\tilde{h}_j f_j)^{\tilde{\rho}} \right)^{\hat{\rho}/\tilde{\rho}} \right)^{1/\hat{\rho}}$$

where  $f_j$  is the consumption of the variety of unaffected services provided by agent  $j$  and  $e_i$  is consumption of a variety of entertainment services as before.

What will be the effect of an increase in  $B$  (the limit of IRS for entertainers) on the incomes in the unaffected occupation? Instead of the formal analysis for this extended model, we only provide the intuition here. An increase in  $B$  impacts the unaffected occupation in the following two ways.

1. The price effect: entertainment services become relatively cheaper. As is typical in a consumers’ decision problem, the price reduction generates two conflicting effects on the spending of each agent on unaffected services: a negative substitution effect and a positive income effect. For the CES case that we are considering, if  $\hat{\rho}$  is positive, then the negative substitution effect dominates the positive income effect and the workers in the unaffected occupation are adversely affected. If  $\hat{\rho}$  equals zero (the Cobb-Douglas case), then these two effects exactly offset each other and these workers are not affected. Finally, the net effect will be positive if the unaffected services and affected ones are complements ( $\hat{\rho} < 0$ ).<sup>21</sup>

2. The aggregate income effect: aggregate income may increase or decrease with the increase in  $B$ , which may then impact unaffected workers positively or negatively.

In addition, we can derive in parallel that a worker with higher human capital in the unaffected occupation acquires a greater share of aggregate spending on unaffected services.

Finally, note that if  $B$  is unchanged then there is no “affected occupation”. Thus an increase in  $A$  affects both occupations in the same way as explained above.

## 5. Empirical Patterns

In this section we bring the theoretical results derived in Section 4 to the data. Specifically, we exploit U.S. occupational data over the last three decades in order to explore the predictions of the model with respect to an increase in the IRS limit,  $B$ .

### 5.1. The Internet as a Rise in the IRS Limit

Since an increase in the IRS limit manifests as an outward shift in workers’ scale of operation, the advent and spread of the internet is an excellent test case for this phenomenon. To do this we focus on the U.S. in order to exploit detailed annual data on workers’ hours and earnings. Specifically, we investigate whether the recent growth in U.S. inequality can in part be attributed to the interaction of distinct occupational features with new information and communication technologies (ICT) – specifically, the internet – as predicted by the model. Throughout, we exploit data on wages and employment within U.S. occupations from the U.S. Current Population Survey (CPS) over the years 1985 to 2006.<sup>22</sup>

In the model,  $B$  represents the limit of IRS for an occupation, reflecting the scale of operation of the workers in the occupation. Differences in the scale of operation across occupations and over time may arise for many reasons; here we argue that an important difference arose due to the

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<sup>21</sup>By (3), if the price of entertainment services,  $P$ , decreases, the fraction spent on the subsistence good,  $\frac{1}{1+\mu^{1/(\hat{\rho}-1)}P^{\hat{\rho}/(\hat{\rho}-1)}}$ , decreases too, unless  $\hat{\rho} \leq 0$  (but we assume  $\hat{\rho} \geq 0$  – namely, that the subsistence good and services are substitutes).

<sup>22</sup>The data were obtained from IPUMS (see Ruggles, Alexander, Genadek, Goeken, Schroeder and Sobek (2010))

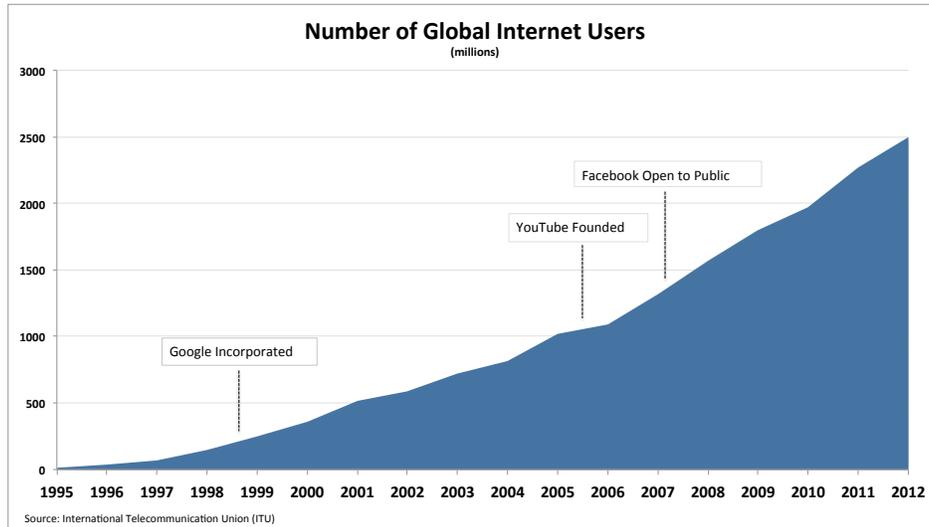


Figure 9: Growth in Global internet Access

rapid expansion of the internet beginning in the mid-1990s, which differentially affected the scale of operation for different occupations. For instance, with the expansion of the internet musicians and online retailers could sell to more customers, whereas the number of customers that a barber or dentist could sell to remained effectively unchanged. In the language of the model these were affected and unaffected occupations, respectively.<sup>23</sup>

Formally, we construct a measure of the extent to which the output of each of 341 U.S. occupations generated internet sales over the period 1985 to 2006, which clearly includes the period over which the internet has become widely accessible – approximately 1995 on, see Figure 9 – as well as the decade prior to this period, which will allow us to control for potential confounding factors. Since this measure reflects the component of the limit of IRS that is due to the internet, we refer to it as  $B^{Int}$ , and we define it in the following way:

$$B_{it}^{Int} = \sum_j (IntShr_{jt} \times OccShr_{ijt}) \quad (26)$$

where  $IntShr_{jt}$  is the share of industry  $j$  sales in year  $t$  that was made over the internet and  $OccShr_{ijt}$  is the share of occupation  $i$ 's total hours employed in industry  $j$  in year  $t$ . Thus, the latter term reflects the importance of each industry, in terms of labor hours, to each occupation, while the former term captures the extent to which firms within each industry sell their output over

<sup>23</sup> Among the many reasons that the magnitude of the effect would have differed across occupations are the fact that occupation output may be more or less easily digitized and, therefore, transmitted electronically, or that language may be a key determinant of demand for the occupation output, limiting demand to markets that share the language – e.g., marketing occupations.

the internet.<sup>24</sup> Of course, the measure may not perfectly capture the extent to which occupational services are linked to internet sales. For instance, even within an industry that sells a substantial amount over the internet, some occupations may be specialized in brick-and-mortar sales, while others are focused on internet sales. Here we assume each occupation’s output is allocated according to the share of sales that occur over the internet within an industry. Furthermore, our analysis below will focus on the implications for wages, but the elasticity of occupational wages to internet sales may vary across occupations for many reasons, from which we abstract. Nevertheless, we believe the measure is a reasonable one, and captures well the differential extent to which the market for occupational services grew over the period due to the expansion of the internet. Table 1 lists the top 20 (left column) and bottom 20 (right column) occupations in terms of their exposure to the internet according to this measure, where the measure itself ranges from 0 (completely unexposed) to 0.28 (most exposed).

1	Financial services sales occupations	317	Batch food makers
2	Motion picture projectionists	318	Miners
3	Cabinetmakers and bench carpenters	319	Dental assistants
4	Editors and reporters	320	Pest control occupations
5	Furniture and wood finishers	321	Managers of medicine and health occupations
6	Typesetters and Compositors	322	Primary school teachers
7	Other financial specialists	323	Mail carriers for postal service
8	Broadcast equipment operators	324	Postal clerks, excluding mail carriers
9	Computer Software Developers	325	Special education teachers
10	Actors, directors, producers	326	Secondary school teachers
11	Nail and tacking machine operators	327	Legislators
12	Upholsterers	328	Clergy and religious workers
13	Advertising and related sales jobs	329	Inspectors of agricultural products
14	News vendors	330	Welfare service aides
15	Industrial Engineers	331	Postmasters and mail superintendents
16	Designers	332	Meter readers
17	Sawing machine operators and sawyers	333	Mail and paper handlers
18	Proofreaders	334	Hotel clerks
19	Writers and authors	335	Judges
20	Supervisors and proprietors of sales jobs	336	Sheriffs, bailiffs, correctional institution officers

Table 1: Top and Bottom 20 Occupations by internet Exposure

### *IRS, the Internet and Wage Inequality*

In this subsection we test the implications of an increase in  $B$  for the employment and wage distribution within occupations. We begin with the effect on the income distribution where we test whether 1) lower-end workers see declining incomes as Proposition 3 indicates and 2) the change in a worker’s income is rising in her initial income, as Proposition 4 indicates. To do this, we first “clean” the log wage of demographic (gender, race), educational attainment, and experience (age,

<sup>24</sup>internet sales by industry come from Census’ E-Stats database, available at <http://www.census.gov/econ/estats/>. See Appendix D for details regarding the construction of the measure.

age squared) in a first-stage regression in order to focus as much as possible on wage variation that arises due to the intrinsic features of the occupations, rather than changes in the composition of the workforce. In addition, to the extent that the internet led to greater IRS due to industry-specific, time-invariant features, rather than occupation features, we would like to remove this variation, and we do so by including industry fixed effects in the first stage. The residuals from this regression serve as the relevant wage variation going forward.<sup>25</sup>

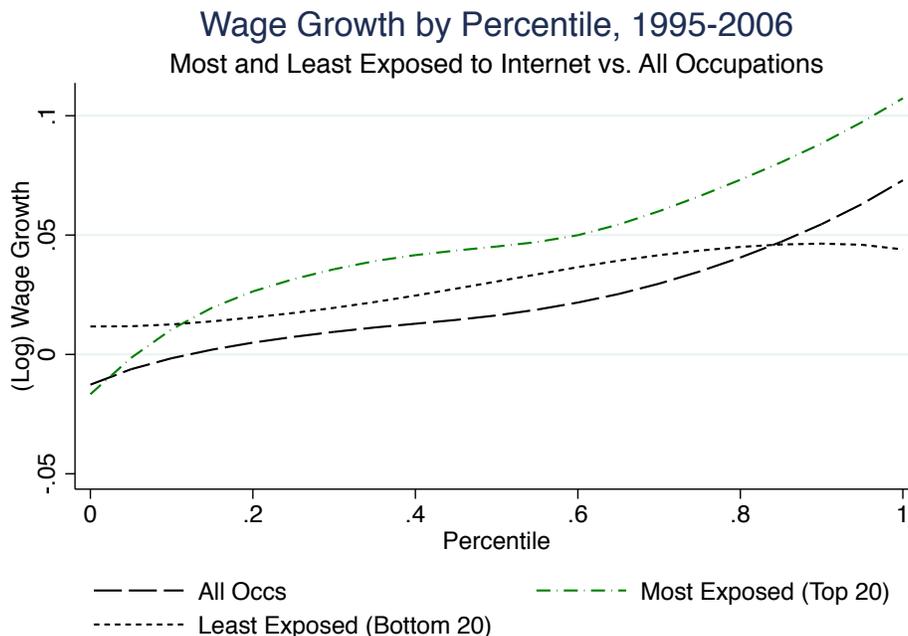


Figure 10: Wage Growth Across the Wage Distribution, Most and Least Affected by the internet

Figure 10 provides some initial evidence in support of the Propositions. We compare log (cleaned) wage growth from 1995 to 2006 across the wage distribution (defined at the beginning of the period) for the most and least internet-affected occupations according to (26). The Figure shows that for the top 20 most affected occupations workers at the low end of the income distribution suffered a loss in income over the period, but this is *not* true of workers at the low end of the distribution within the least affected occupations, consistent with Proposition 3. At the same time, the relationship between income growth and initial income was much “steeper” over the period within the most affected occupations relative to the least affected occupations, as predicted by Proposition 4. In other words, income inequality grew to a much greater extent within these occupations.<sup>26</sup>

We next move toward a more formal test of Proposition 4 by exploiting information on all U.S. occupations, comparing the change in inequality within affected occupations relative to unaffected

<sup>25</sup>Note that we deal with top-coding in the CPS by following the method described in Bakija, Cole and Heim (2010). We also perform all the regressions with the highest earners removed, finding nearly identical results.

<sup>26</sup>The same pattern holds for slightly larger or smaller sets of occupations – e.g., the top 15 or 25.

occupations due to the spread of the internet. In doing so, we need to consider not only the implications of a rise in  $B$  for the *affected* occupations, but also the general equilibrium effects on the *unaffected* occupations, which, as noted in Section 4.3, work via two channels. First, aggregate income may rise which will raise income inequality due to the fact that workers with more human capital – who thus earn more – gain a relatively large share of the income growth across all occupations. However, this effect will be the same across both affected and unaffected occupations and so will be absorbed in the included time fixed effects. Second, the relative price of the output produced by affected occupations will fall, and this will reduce (increase) spending on the unaffected occupation output when the two outputs are substitutes (complements) for one another. We believe that the outputs of different occupations are, in general, gross substitutes on average, though there are certainly instances of complementarity among some sets of occupations. This implies that unaffected occupations will see falling inequality due to a rise in  $B$ . This effect strengthens our prediction that affected occupations should see rising inequality relative to unaffected occupations – in terms of a greater  $d\Delta w/dw$  – due to an increase in  $B$ .

We formally test this prediction by estimating the following specification:

$$\Delta Wage_{qi,t:t+1} = c + \beta_1 \Delta B_{it:t+1}^{Int} + \beta_2 Wage_{qit} + \beta_3 (\Delta B_{it:t+1}^{Int} \times Wage_{qit}) + \sigma_i t + \delta_i + \alpha_t + \epsilon_{qit} \quad (27)$$

where  $\Delta Wage_{qit}$  is the annual change in the log wage in an occupation  $i$  at wage vigintile (20-quantile)  $q$  in year  $t$ ,  $\Delta B_{it}^{Int}$  is the regressor of interest described previously in annual changes, and  $\sigma_i t$  are linear occupation-specific trends.<sup>27</sup> Finally,  $\delta_i$  and  $\alpha_t$  are occupation and year fixed effects and  $\epsilon_{qit}$  is a disturbance term. Note that the inclusion of occupation fixed effects implies that we focus narrowly on the differential inequality growth within occupations over the period. Moreover, the occupation-specific trends will control for the common component of wage growth across percentiles within an occupation over the pre- and post-internet periods. These may be important if, for instance, occupations with rising average wages were the most likely to invest in internet technologies, a plausible scenario. Again, Proposition 4 predicts that  $\beta_3 > 0$ .

This identification strategy leaves open the possibility that there are omitted variables that will bias our estimates of  $\beta_3$ . Most problematic are those that are both correlated with the intensity of internet sales across occupations while also directly increasing wage inequality in those occupations for reasons outside the model. In particular, the rapid fall in the price of computing technologies over the period, which were differentially adopted across industries and occupations while also facilitating access to the internet, may have directly increased wage inequality within affected occupations,

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<sup>27</sup>We tried specifications that included quadratic and cubic trends, which have little additional explanatory power. The results are also qualitatively invariant to defining the distribution with more or fewer quantiles. Results are available upon request.

Table 2: Differential Wage Impact Due to Internet Exposure

	(1)	(2)	(3)	(4)	(5)
	Wage Growth	Wage Growth	Wage Growth	Wage Growth	Wage Growth
Internet Exposure	0.0291 (0.0683)	0.00807 (0.0594)	0.00206 (0.0608)	0.0134 (0.0606)	0.113 (0.127)
Base Wage		-0.0867*** (0.00428)	-0.0871*** (0.00429)	-0.0824*** (0.00425)	-0.0822*** (0.00483)
Base Wage x Internet Exp		0.565*** (0.218)	0.520** (0.218)	0.449** (0.226)	0.391 (0.429)
Occ Trend			-0.000*** (0.000)		
Computer Exp, 85-94				0.011*** (0.00364)	
Computer Exp, All Years					0.0218 (0.015)
Observations	144647	144647	144647	138503	75129
Occ FE	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes

Standard errors in parentheses are clustered at the occupation level.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

independent of any effect via access to the internet. In this case, our estimates will be biased upward – i.e., we will over-estimate the differential impact of the internet on our occupation groups.

Unfortunately, any attempt to control for contemporaneous computer use will be hampered by the fact that much of the variation we are interested in – i.e., the variation due to occupation-specific sales over the internet – will be highly correlated with computer use itself. As a result, we present two specifications that attempt to control for variation in computer use across and within occupations. First, in our preferred specification we control for occupation-specific computer use in the years just prior to the internet period, from 1989 to 1994, using data from the CPS computer use supplements.<sup>28</sup> Our measure is the share of hours worked in an occupation by workers who use a computer. By focusing on the pre-internet period only, we exploit variation in computer use that is unrelated to the internet but can potentially explain future wage inequality growth. In our second specification we simply control for computer use throughout the period.

Table 2 column (2) presents the results of our primary specification, after which we progressively add occupation-specific trends (column (3)), controls for pre-internet computer use (column (4)) and computer use in all years (column (5)). The results provide fairly strong evidence that wage inequality growth was greater in occupations whose services were more likely to be sold over the internet, with all specifications generating significant, positive coefficients on the interaction term,

<sup>28</sup>This survey asks respondents whether they “directly use a computer at work”. The data source is again Ruggles et al. (2010).

except the final specification. Column (1), which excludes the interaction term, only provides weak evidence that wage growth was on average larger in internet-exposed occupations, suggesting that the primary effect of the internet was to widen inequality within occupations, with little effect on the mean wage.

The estimates in columns (4) and (5) are consistent with our hypothesis that variation in computer use across industries is to some extent co-linear with internet exposure in our sample. Specifically, when controlling for computer use throughout the period the impact of internet sales on wage inequality is diminished, and not significant. Nevertheless, the positive coefficient is suggestive of an impact of internet exposure above and beyond that which occurs directly via the use of computers within an occupation, suggesting that it is not *only* occupational computer use that is driving the estimated inequality trends indicated in Table 2.

Table 3: Differential Employment Impact Due to Internet Exposure

	(1) Emp Growth	(2) Emp Growth	(3) Emp Growth	(4) Emp Growth
Internet Exposure	-0.369 (0.348)	-0.369 (0.348)	-0.495 (0.358)	-0.405 (0.332)
Occ Trend		-0.000* (0.000)	-0.000 (0.000)	-0.000 (0.000)
Computer Exp, 85-94			-0.001 (0.003)	
Computer Exp, All Years				0.001 (0.005)
Observations	9061	9061	8347	4610
Year FE	Yes	Yes	Yes	Yes

Standard errors in parentheses are clustered at the occupation level.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

As a test of Proposition 2, which states that an expansion in  $B$  will reduce employment within affected occupations, Table 3 presents the results of a similar, but simpler, set of regressions in which now annual employment growth in an occupation is the dependent variable. To construct the employment growth measure we again “clean” the variation in log employment of demographic, education and industry-specific variation and then take the difference across years. Formally, we estimate:

$$\Delta Emp_{i,t:t+1} = c + \tau_1 \Delta B_{it}^{Int} + \gamma_i t + \alpha_t + \epsilon_{it} \quad (28)$$

where the regressors are as described above and due to the first-differencing we are again focused on within-occupation variation. Table 2 presents the results, where column (2) adds occupation-specific trends and columns (3) and (4) add the controls for computer use within an occupation. The estimates are negative in all cases, as the model predicts, but are not significant. Overall we

take this as mild evidence that internet-affected occupations shrunk relative to other occupations during the internet period, consistent with Proposition 2.

Finally, since year-to-year variation in hours worked and wages can be noisy within the CPS at the occupation level, in Appendix E we present the results of regressions that are identical to (27) and (28) except they are estimated in long differences. The estimated effects are qualitatively the same, and much more precise.

## 6. Concluding Remarks

Technological changes of various types are constantly reshaping economies, generating winners and losers. In this paper we have introduced two new channels through which technological change may increase within-occupation income inequality. In both cases, the way in which tasks are performed within an occupation remains unchanged, and yet the income distribution within the occupation increases as a result of the technological change.

The first channel that we have considered arises due to an increase in the scale of operation for certain occupations. In this case the model's mechanisms are unique: the rise in income inequality that results from the increase in scale of operation occurs due to increased competition among workers, which drives workforce reallocation within the occupation and redistributes revenue across the workers. We model this type of technological change as an increase in the limit up to which the production of the affected occupation displays Increasing Returns to Scale. This generates two conflicting effects. On the one hand, the scale of operation for workers within the occupation increases, which benefits them. On the other hand, since this is true for all workers within the occupation, they therefore face fiercer competition for their services. The latter effect is felt equally by all workers, but the benefits are greater for workers with relatively more human capital, who charge higher prices. The net effect is therefore to increase inequality within the occupation. Moreover, lower-end workers are forced out of the human-capital-intensive occupations into unskilled work.

The second channel arises due to any increase in labor productivity, and therefore aggregate income. In the model, workers with greater human capital acquire a larger fraction of aggregate income by providing better quality services, and thus a larger fraction of the increase in aggregate income accrues to them. This again increases income inequality.

Finally, we test our model's predictions using U.S. data, finding patterns in the data consistent with the mechanisms we highlight.

We conclude by noting that there are clearly many other types of technological changes, and each may have different implications for the economy. Furthermore, there are a range of forces, both technological and otherwise, that have contributed to the rising inequality observed in many countries in recent decades. We believe that the technological forces that we consider here are

important in part due to their near ubiquity, as well as the fact that they may be relatively difficult for policy-makers to counter compared to institutional factors, such as the extent of unionization or tax policies.

Tianxi Wang, Department of Economics, University of Essex

Greg Wright, Department of Economics, University of California, Merced

## Appendix A. Proofs

### Appendix A.1. Proof of Proposition 2

*Proof.*  $k$  is determined by equation (21). Differentiating with respect to  $B$  on both sides, we find

$$[d(\mu^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\hat{\rho}-\rho}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}})/dk - (A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] \cdot dk/dB = (k-1)d(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}/dB + d[(1-k_0)(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] / dB$$

We further know that  $d(\frac{1}{1-\hat{\rho}} H_k^{\frac{\hat{\rho}-\rho}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}})/dk < 0$  because  $dH_k/dk < 0$  and  $\frac{\hat{\rho}-\rho}{1-\hat{\rho}} > 0$ , and  $dh_k/dk > 0$ . Therefore, on the LHS of the equation the term in front of  $dk/dB$  is negative.

On its RHS,  $d(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}/dB > 0$  and  $k-1 < 0$ . Therefore, if  $d[(1-k_0)(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] / dB \leq 0$ , which (as  $k_0 = \frac{Bc}{A+Bc}$ ) is equivalent to  $c \geq \frac{\hat{\rho}}{1-\hat{\rho}} \cdot \frac{A}{B}$ , then the RHS is negative and thus  $dk/dB > 0$ . ■

### Appendix A.2. Proof of Proposition 3

*Proof.* We need only show that  $dm_i/dB < 0$  for  $i = k$ . When this is the case, the Proposition follows from the fact that  $dm_i/dB$  is continuous in  $i$ . By (22),  $\frac{dm_i}{dB} = h_i^\rho/h_k^\rho \cdot [c - (A+Bc) \cdot \rho \cdot (\log h_k)'] \cdot \frac{dk}{dB} - c$ . At  $i = k$ , therefore,  $\frac{dm_i}{dB} = c - (A+Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB} - c = -(A+Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB} < 0$  because  $(\log h_k)'$  is assumed to be positive and  $\frac{dk}{dB} > 0$  by Proposition 2. ■

### Appendix A.3. Proof of Proposition 4

*Proof.* From (22),  $m_i$  increases with  $h_i^\rho$ . Therefore, to prove the proposition, it suffices to prove that  $dm_i/dB$  increases with  $h_i^\rho$ . By (22),

$$\frac{dm_i}{dB} = h_i^\rho/h_k^\rho \cdot [c - (A+Bc) \cdot \rho \cdot (\log h_k)'] \cdot \frac{dk}{dB} - c. \quad (\text{A.1})$$

Only the first term depends on  $h_i$ . Therefore,  $dm_i/dB$  increases with  $h_i^\rho$  if and only if  $c - (A+Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB} > 0 \Leftrightarrow$

$$c > (A+Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB}.$$

The identity of the marginal entertainer,  $k$ , is determined by equation (21). Taking the logarithm of both sides:  $\frac{1}{1-\hat{\rho}} \log \mu + \frac{\hat{\rho}-\rho}{\rho(1-\hat{\rho})} \log H_k^\rho - \frac{\rho}{1-\hat{\rho}} \log h_k = \log(k-k_0) - \frac{\hat{\rho}}{1-\hat{\rho}} \log(A/B + c)$ . Now taking the derivative with respect to  $B$  on both sides and noting that  $\frac{dH_k^\rho}{dk} = -h_k^\rho$  and recalling  $k_0 = \frac{Bc}{A+Bc}$ :  $[-\frac{\hat{\rho}-\rho}{\rho(1-\hat{\rho})} h_k^\rho/H_k^\rho - \frac{\rho}{1-\hat{\rho}} (\log h_k)'] \cdot \frac{dk}{dB} = \frac{1}{k-k_0} \cdot [\frac{dk}{dB} - \frac{Ac}{(A+Bc)^2}] + \frac{\hat{\rho}}{1-\hat{\rho}} \cdot \frac{A}{(A+Bc)B} \Rightarrow$

$$\frac{dk}{dB} = \frac{1/(k-k_0) \cdot Ac/(A+Bc)^2 - \hat{\rho}/(1-\hat{\rho}) \cdot A/[A+(Bc)B]}{1/(k-k_0) + \frac{\rho}{1-\hat{\rho}} (\log h_k)' + \frac{\hat{\rho}-\rho}{\rho(1-\hat{\rho})} h_k^\rho/H_k^\rho}.$$

The numerator is smaller than  $1/(k-k_0) \cdot Ac/(A+Bc)^2$ , while the denominator is greater than

$1/(k - k_0) + \frac{\rho}{1-\rho}(\log h_k)'$ , which is in turn greater than  $1/(k - k_0) + \rho(\log h_k)'$ . Therefore,

$$\frac{dk}{dB} < \frac{Ac/(A + Bc)^2}{1 + \rho(\log h_k)'(k - k_0)}.$$

With this inequality, the inequality (A.2) follows from  $c > (A + Bc) \cdot \rho \cdot (\log h_k)'$ .  $\frac{Ac/(A+Bc)^2}{1+\rho(\log h_k)'(k-k_0)}$ , which, with rearrangement and noting that  $k_0 = \frac{Bc}{A+Bc}$ , is equivalent to:

$$\frac{\rho \cdot h'_k/h_k}{1 + \rho \cdot h'_k/h_k \cdot (k - k_0)} < \frac{1}{1 - k_0}.$$

Note the LHS of the inequality increases with  $\rho$  and  $\rho \leq 1$ . The inequality, therefore, follows from

$$\frac{h'_k/h_k}{1 + h'_k/h_k \cdot (k - k_0)} < \frac{1}{1 - k_0},$$

which follows from assumption (12) as  $k > k_0$ . ■

#### Appendix A.4. Proof of Lemma 1

*Proof.* By (A.1),  $\frac{dm_i}{dB} > 0$  if

$$h_i^\rho/h_k^\rho \cdot [c - (A + Bc) \cdot \rho \cdot (\log h_k)'] \cdot \frac{dk}{dB} > c. \quad (\text{A.2})$$

With an upper bound of  $\frac{dk}{dB}$  given by (A.2), this inequality follows from:  $h_i^\rho/h_k^\rho \cdot [c - (A + Bc) \cdot \rho \cdot (\log h_k)'] \cdot \frac{Ac/(A+Bc)^2}{1+\rho(\log h_k)'(k-k_0)} > c \Leftrightarrow$

$$h_i^\rho/h_k^\rho \cdot \left[1 - \frac{A}{A + Bc} \cdot \frac{\rho \cdot (\log h_k)'}{1 + \rho(\log h_k)'(k - k_0)}\right] > 1, \quad (\text{A.3})$$

which is equivalent to (24). ■

#### Appendix A.5. Proof of Lemma 2

*Proof.* We prove the lemma in three steps.

Step 1: If  $h_1 > 1$ , then

$$k - k_0 < D \left(\frac{h_1}{h_k}\right)^{\frac{\rho}{1-\rho}} (1 - k)^{\frac{\rho-\hat{\rho}}{\rho(1-\hat{\rho})}}. \quad (\text{A.4})$$

*Proof:*  $k$  is determined by equation (21), or equivalently,  $k - k_0 = DH_k^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}}$ . Note that  $H_k = \{\int_k^1 h_i^\rho\}^{\frac{1}{\rho}} |_{h'_i > 0} < \{\int_k^1 h_1^\rho\}^{\frac{1}{\rho}} = h_1(1 - k)^{\frac{1}{\rho}}$ . Therefore,  $H_k^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}} = \left(\frac{H_k^{\rho-\hat{\rho}}}{h_k^\rho}\right)^{\frac{1}{1-\hat{\rho}}} < \left(\frac{h_1^{\rho-\hat{\rho}}(1-k)^{\frac{\rho-\hat{\rho}}{\rho}}}{h_k^\rho}\right)^{\frac{1}{1-\hat{\rho}}} = h_1^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}} / h_k^{\frac{\rho}{1-\hat{\rho}}} \cdot (1 - k)^{\frac{\rho-\hat{\rho}}{\rho(1-\hat{\rho})}} |_{\frac{\rho-\hat{\rho}}{1-\hat{\rho}} \leq \frac{\rho}{1-\hat{\rho}}} \text{ and } h_1 > 1 < h_1^{\frac{\rho}{1-\hat{\rho}}} / h_k^{\frac{\rho}{1-\hat{\rho}}} \cdot (1 - k)^{\frac{\rho-\hat{\rho}}{\rho(1-\hat{\rho})}}$ , which implies (A.4).

Step 2:

$$k < f(k_0, D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\hat{\rho}}}). \quad (\text{A.5})$$

*Proof:* Let  $\tau := f(k_0, D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\hat{\rho}}})$ . By the definition of  $f(\cdot, \cdot)$ ,  $\tau - k_0 = D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\hat{\rho}}} \cdot (1 - \tau)^{\frac{\rho - \hat{\rho}}{\rho(1-\hat{\rho})}}$ . The two sides of this inequality minus, respectively, the two sides of inequality (A.4) leads to  $\tau - k > D(\frac{h_1}{h_k})^{\frac{\rho}{1-\hat{\rho}}} [(1 - \tau)^{\frac{\rho - \hat{\rho}}{\rho(1-\hat{\rho})}} - (1 - k)^{\frac{\rho - \hat{\rho}}{\rho(1-\hat{\rho})}}]$ . This inequality can hold true only if  $\tau > k$ : if  $\tau \leq k$ , then the LHS of the inequality is negative, while the RHS is positive – and thus cannot be strictly smaller than the LHS – because  $1 - \tau \geq 1 - k$ , which implies  $(1 - \tau)^{\frac{\rho - \hat{\rho}}{\rho(1-\hat{\rho})}} - (1 - k)^{\frac{\rho - \hat{\rho}}{\rho(1-\hat{\rho})}} \geq 0$  (as  $\frac{\rho - \hat{\rho}}{\rho(1-\hat{\rho})} > 0$ ). Q.E.D.

Step 3: We prove the Lemma by showing that  $\zeta \geq h_1/h_k$  leads to a contradiction. Clearly,  $f(k_0, y)$  increases with  $y$ , and therefore if  $\zeta \geq h_1/h_k$ , then  $f(k_0, D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\hat{\rho}}}) < (f(k_0, D \cdot \zeta^{\frac{\rho}{1-\hat{\rho}}}))$ , which together with (A.5) implies that  $k < f(k_0, D \cdot \zeta^{\frac{\rho}{1-\hat{\rho}}})$ . Since  $h'(i) > 0$ , then  $h_k < h(f(k_0, D \cdot \zeta^{\frac{\rho}{1-\hat{\rho}}}))$ . Thus we have

$$\zeta \geq \frac{h_1}{h_k} > \frac{h_1}{h(f(k_0, D \cdot \zeta^{\frac{\rho}{1-\hat{\rho}}}))},$$

which implies  $\zeta \cdot h(f(k_0, D \cdot \zeta^{\frac{\rho}{1-\hat{\rho}}})) > h_1$ , in contradiction to the lemma. Q.E.D. ■

## Appendix A.6. Proof of Proposition 6

*Proof.*  $k$  is determined by equation (21). Differentiate with respect to  $A$  on both sides, and we find

$$[d(\mu^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\rho - \hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}})/dk - (A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] \cdot dk/dA = (k - 1)d(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}/dA + d[(1 - k_0)(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] / dA$$

We saw  $d(\mu^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\rho - \hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}})/dk < 0$  because  $dH_k/dk < 0$  and  $\frac{\rho - \hat{\rho}}{1-\hat{\rho}} > 0$ , and  $dh_k/dk > 0$ . Therefore, on the LHS of the equation the term in front of  $dk/dA$  is negative.

On its RHS,  $d(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}/dA < 0$  and  $k - 1 < 0$ . Therefore, if  $d[(1 - k_0)(A/B + c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] / dA \geq 0$ , which (as  $k_0 = \frac{Bc}{A+Bc}$ ) is equivalent to  $c \geq \frac{\hat{\rho}}{1-\hat{\rho}} \cdot \frac{A}{B}$ , then the RHS is positive and thus  $dk/dA < 0$ . ■

## Appendix A.7. Proof of Proposition 7

*Proof.* By (22),  $\frac{dm_i}{dA} = \frac{h_i^\rho}{h_k^\rho} + (Bc + A)(-\rho) \frac{h_i^\rho}{h_k^{\rho+1}} \cdot h'_k \cdot \frac{dk}{dA} = \frac{h_i^\rho}{h_k^\rho} \cdot [1 + (Bc + A)(\log h_k)' \cdot (-\rho \frac{dk}{dA})] |_{-\rho \frac{dk}{dA} > 0}$  (by Prop. 5)  $> \frac{h_i^\rho}{h_k^\rho} \geq 1$ . Moreover, by (22),  $\frac{h_i^\rho}{h_k^\rho} = \frac{m_i + Bc}{A + Bc}$ . Then,  $\frac{dm_i}{dA} = \frac{m_i + Bc}{A + Bc} \cdot [1 + (Bc + A)(\log h_k)' \cdot (-\rho \frac{dk}{dA})]$  and increases with  $m_i$ . ■

## Appendix B. An Example in which $dm_i/dB$ Decreases with $m_i$

Following the proof of Proposition 3,  $dm_i/dB$  decreases with  $m_i$  if

$$c < (A + Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB}. \quad (\text{B.1})$$

To construct such an example, we therefore want  $(\log h_k)'$  to be large enough. Here is one example. Let  $\hat{\rho} = 0$  and let the distribution of human capital be given by

$$h_i = \left\{ \begin{array}{l} x \text{ if } i < k - \epsilon/2 \\ x + \frac{\delta}{\epsilon}(i - k + \epsilon/2) \text{ if } k - \epsilon/2 \leq i \leq k + \epsilon/2 \\ x + \delta \text{ if } k + \epsilon/2 < i \end{array} \right\}$$

for some  $\epsilon, \delta, k > 0$  and  $k - \epsilon/2 > 0$  and  $k + \epsilon/2 < 1$ . Therefore,  $h'_k = \frac{\delta}{\epsilon}$  and  $h'_k/h_k \rightarrow \infty$  if  $\epsilon \rightarrow 0$ . For the time being,  $k$  is just a parameter. But this parameter identifies the marginal agent in equilibrium if it satisfies equation (21), which, since  $\hat{\rho} = 0$ , becomes

$$\mu H_k^\rho h_k^{-\rho} = k - k_0. \quad (\text{B.2})$$

With  $\epsilon \rightarrow 0$ , the LHS of this equation approaches  $\mu \frac{(x+\delta)^\rho(1-k)}{(x+\delta/2)^\rho}$ . Thus, with  $\epsilon \rightarrow 0$ ,  $k$  approaches the root of

$$\mu \frac{(x + \delta)^\rho}{(x + \delta/2)^\rho} (1 - k) = k - k_0,$$

denoted by  $\tilde{k}$ . Clearly,  $\tilde{k} < 1$ .

By (A.2), with  $\hat{\rho} = 0$  and some rearrangement

$$\frac{dk}{dB} = \frac{Ac/(A + Bc)^2}{1 + \rho(\log h_k)' \cdot (k - k_0) + h_k^\rho/H_k^\rho \cdot (k - k_0)}. \quad (\text{B.3})$$

From (B.2) it follows that  $h_k^\rho/H_k^\rho \cdot (k - k_0) = \mu$ . Substituting this into (B.3),

$$\frac{dk}{dB} = \frac{Ac/(A + Bc)^2}{1 + \mu + \rho(\log h_k)' \cdot (k - k_0)}.$$

Then, (B.1) is equivalent to

$$\frac{1}{1 - k_0} < \frac{\rho \cdot (\log h_k)'}{1 + \mu + \rho(\log h_k)' \cdot (k - k_0)}, \quad (\text{B.4})$$

where we also apply  $1 - k_0 = \frac{A}{A+Bc}$ . Note that for the RHS of this inequality, if  $\epsilon \rightarrow 0$ ,  $(\log h_k)' \rightarrow \infty$  and  $k \rightarrow \tilde{k} < 1$ , and then the RHS approaches  $\frac{1}{k - k_0} > \frac{1}{1 - k_0}$ , the LHS. Therefore, if  $\epsilon$  is close enough

to 0, inequality (B.4), and thus inequality (B.1), holds true, which means that  $dm_i/dB$  decreases with  $m_i$ .

### Appendix C. Decomposition of Wage Inequality Within and Between Occupations

Using occupational wage data from the U.S. American Community Survey (ACS) we decompose total wage inequality into its between- and within-occupation components (a similar exercise is performed by Akerman, Helpman, Itskhoki, Muendler and Redding (2013)). Since the occupational classification changed in 2003, we limit the decomposition to the period 2003-2010 (we perform the decomposition for each year separately). Formally, we calculate:

$$\frac{1}{N_t} \sum_i (w_{it} - \bar{w}_t)^2 = \frac{1}{N_t} \sum_l \sum_{i \in l} (w_{it} - \bar{w}_{lt})^2 + \frac{1}{N_t} \sum_l N_{lt} (\bar{w}_{lt} - \bar{w}_t)^2 \quad (\text{C.1})$$

where workers are indexed by  $i$  and the year by  $t$ ;  $l$  represents occupations;  $N_{lt}$  and  $N_t$  represent the number of workers in each occupation and overall; and  $w_{it}$ ,  $\bar{w}_{lt}$  and  $\bar{w}_t$  are the log worker wage, the average log occupational wage, and the overall average wage. In using the log wage we ensure the values are independent of the wage units. The first term on the right hand side reflects the within-occupation component of wage inequality.

### Appendix D. Internet Exposure Measure

Our measure of internet exposure,  $B_{it}^{Int}$ , is constructed as in (26). The industry internet sales data come from Census' E-Stats database, which provides the data at the two- and three-digit North American Industry Classification System (NAICS) level. We then concord these to the Ind1990 classification used in the CPS using a straightforward concordance provided by Census. One nuance is that some of the sales data is classified under the industry "E-Merchants" (NAICS 4541) by product, in categories such as Books and Magazines, Music and Videos, etc. We therefore match these to the relevant Ind1990 industries manually. The final step is to calculate (26).

### Appendix E. Wage and Employment Regressions, Long Differences

Here we present the results of regressions identical to (27) and (28) except they are estimated as stacked long differences, covering 1985 to 1994 and 1995 to 2006. In both regressions we present a specification in which we control for computer use in the pre-internet period.

Table E.4: Differential Wage Impact on Occupations Due to Internet Exposure

	(1)	(2)	(3)	(4)
	Wage Growth	Wage Growth	Wage Growth	Wage Growth
Internet Exposure	0.0411 (0.0264)	0.0378 (0.0268)	0.0379 (0.0273)	0.0348 (0.0264)
Base Wage		-0.00325*** (0.00103)	-0.00325*** (0.00102)	-0.00298*** (0.00104)
Base Wage x Internet Exposure		0.208*** (0.0440)	0.208*** (0.0441)	0.205*** (0.0437)
Occ Trend			0.000 (0.000)	0.000 (0.000)
Computer Exp, 85-94				0.0125 (0.009)
Observations	9537	9537	9537	9227
Occ FE	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes

Standard errors in parentheses are clustered at the occupation level.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

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Table E.5: Differential Employment Impact on Occupations Due to Internet Exposure

	(1)	(2)	(3)	(4)
	Emp Growth	Emp Growth	Emp Growth	Emp Growth
Internet Exposure	-0.0417 (0.052)	-0.0417 (0.052)	-0.0429 (0.052)	-0.0592 (0.051)
Occ Trend			-0.000 (0.000)	0.000 (0.000)
Computer Exp, 85-94				0.003 (0.003)
Observations	645	645	645	593
Year FE	Yes	Yes	Yes	Yes

Standard errors in parentheses are clustered at the occupation level.

\*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

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