Increasing Returns to Scale Within Limits: A Model of ICT and Evidence

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Abstract: A key feature of Information and Communication Technologies (ICT) is that they may increase the scale of operation for workers in some occupations. We model the scale of operation as the limit up to which the production technology displays increasing returns to scale. We then explore the implications of this feature of ICT for the income distribution as well as individual's occupational choices. We find that an ICT-induced increase in the scale of operation for workers within an occupation intensifies competition between the workers. This drives the lowest-ability workers out of the occupation, reduces the earnings of the next lowest-ability workers and raises the overall log income gap within the occupation. We compare these effects with those produced by more traditional technological changes that increase the quantity of output per unit of time. Lastly, we test the theoretical results with U.S. data and find support for the predictions.

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1. Introduction

A key feature of the modern world is the rapid advancement of Information and Communications Technologies (ICT), from the early days of radio, to television, and now the internet. Compared to traditional technological progress – such as the invention of the steam engine or factory automation – a unique aspect of the ICT revolution is that new ICT typically enables a given amount of output to generate greater value, whereas "old" technologies typically enlarged the quantity of output that could be produced per unit of time. For example, the invention of the radio allowed a singer's voice to be heard far beyond the walls of a theater, but of course did not allow her to sing a greater number of songs per hour. Similarly, the invention of television allowed a football match to be seen by audiences around the world, without changing the actual performance of footballing tasks.

More recently, the spread of the internet has spawned a "New Economy",[1] in which traditional business models have been upended. At the same time, new and unforeseeable occupations have appeared, such as "unboxing" in which a presenter unwraps a toy in a dramatic way. Critically, only the existence of a platform such as YouTube could provide a large enough audience to make such a business model profitable; without this technology it would be unimaginable that such an occupation would exist. Thus, by allowing entrepreneurs to reap greater value from their products, new ICT has fundamentally altered the marketplace. In this paper we focus on the implications of this feature of ICT progress for the income distribution and individual career choices, highlighting the way in which these implications differ from those due to traditional technological progress.

The importance of workers' scale of operation has not gone unnoticed: a strand of the literature beginning with Rosen (1981) has applied this notion to explain certain features of the income distribution. This literature, however, has not formally modeled an increase in the scale of operation and explored its effects, as we do here. Specifically, we model the scale of operation as the limit up to which the production technology of the workers displays Increasing Returns to Scale. Formally, for occupations in which "scale of operation" is an important determinant of workers' income, we say that the production technology displays Increasing Returns to Scale up to some Limit (IRSL).[2] In the context of the examples above, while it is costly to produce a song with mass appeal, it costs little to admit an additional person into the theater to hear it up to the point that the theater is filled. This feature of large fixed costs and small marginal costs, over some range, is reflective of the presence of IRS up to some limit – i.e., the theater's capacity in the pre-radio era. Similarly, the capacity of the stadium defines the limit of IRS for football players prior to the invention of television, while the number of the children who can fit into a room defines the limit of IRS for the

[1] This term seems to have originated in a *Time* magazine cover story in 1983 that discussed the transition from an industrial economy to a more technologically-oriented service economy. See "The New Economy" by Charles P. Alexander, *Time* magazine, May 30, 1983.

[2] We note that taking the limit to infinity, IRSL subsumes IRS as a special case.

"unboxing" performer prior to the existence of the internet.

Formally, we consider a continuum of agents with equal endowments of unskilled labor but heterogeneous endowments of human capital. They choose to subsist either by employing their labor, or by employing their human capital and thereby becoming a "professional", for which the quality of their output depends on the size of their human capital endowment.[3] Professionals hire labor in order to produce a stream of services and labor is also separately used to produce a subsistence good. Labor is homogenous and its providers compete under perfect competition, whereas the services provided by individual professionals are differentiated. We motivate this setup by noting that labor (i.e., the human body) is a relatively homogenous input: in most instances one individual's labor can substitute for another's. On the other hand, human capital (i.e., the human mind) is much more idiosyncratic and diverse. As a result, professionals produce differentiated output and thus compete via monopolistic competition. After committing her time to supply human capital rather than labor – which entails the payment of a fixed cost – a professional hires labor to produce her variety of services at constant returns to scale up to the technological limit, denoted B. Thus, a professional's production technology displays Increasing Returns to Scale *up to some Limit* (which we denote IRSL), with the limit given by B. This B therefore represents the maximum scale of operation for a professional – for example, the capacity of the theater in which a singer performs. Technological progress that increases B thus increases the maximum, *potential* scale of operation for all professionals.[4] The primary technological shock that we focus on – i.e., our proxy for innovations in ICT – is reflected in an increase in B for some occupation. In addition, we use an increase in A to capture traditional technological progress that increases the quantity of output per unit of time, as A represents the quantity of the subsistence good produced per unit of labor. A comparison in effect for income distribution and career choice between an increase in A and that in B sheds light on how the ICT progress differs the traditional technology.

We show that an increase in either B or A will widen inequality between professionals within an occupation, but via very different mechanisms. Consider first an increase in B, the IRS limit, which is our primary variable of interest. On the one hand, each professional within the occupation can sell more, which benefits them. On the other hand, since the capacity of all professionals increases each of them faces fiercer labor market competition. Whereas the increase in competition is the same for all professionals, the expansion in capacity delivers greater benefits to those who have more human capital and are therefore able to charge a higher price for their stream of services. As a result, more talented professionals reap greater gains from a given increase in B, which therefore

[3]This feature of an identical labor endowment and heterogeneous human capital (or ability) is also found in Lucas Jr (1978) and Monte (2011).

[4]However, this does not necessarily increase the *actual* scale of operation for a particular professional. For example, the internet may enable a singer to access several hundred million potential customers, but the singer will likely be unable to sell to each of these people.

raises income inequality between professionals. Now consider an increase in A. This increases the productivity of labor hired by professionals as well as the wage paid to labor, which results in no net effect on the marginal cost, nor the profit, associated with professionals' production of their stream of occupational services. Nevertheless, an increase in A widens income inequality between professionals due to a general equilibrium effect, with the intuition as follows. First, the primary impact of an increase in A is to raise aggregate income. Second, since professionals with greater human capital acquire a larger fraction of aggregate income by providing better quality services, a larger fraction of the increase in aggregate income accrues to them. This again increases income inequality.

While an increase in either B or A will raise income inequality, the consequences for selection into and out of occupations – what can be thought of as the "career margin" – are quite different. First, an increase in A induces more agents to exploit their human capital relative to their labor, and thereby select into a professional occupation, leading to a greater variety of differentiated output. We note that this effect is consistent with the long-run historical shift away from labor-intensive work and toward human-capital-intensive work, such as the long-run growth in the supply of artists, writers, sportsmen, comedians, etc. On the other hand, an increase in B drives agents *out of* human-capital-intensive professional work by intensifying competition between professionals. For example, the popularity of television makes it difficult to earn a living as a local comedian, while the expansion of the internet has reduced opportunities for travel agents. This result is thus consistent with a range of anecdotal observations on the effects of ICT advancement, and is also consistent with the employment declines among certain occupations that we document in Section 5.

In a final section we test two key predictions of the theory: (1) technological change that increases the scale of operation for workers within an occupation will also increase inequality within the occupation; and (2) this technological change will simultaneously reduce employment in the occupation. For our empirical exercise we exploit the rapid expansion of global internet access beginning in the mid-1990s. We exploit variation across occupations in the extent to which the stream of services associated with each U.S. occupation generated internet sales in each year over the period 1995-2006.[5] An increase in this measure for a particular occupation therefore reflects an increase in the reach, or scale of operation, for those workers – i.e., an increase in the IRS limit. We first illustrate some descriptive facts associated with the internet era, highlighting the fact that within-occupation inequality has been important, particularly for the occupations most exposed to internet sales. Then, in a regression framework we test the relationship between our empirical measure of occupational exposure to internet sales and changes in wage inequality and employment

[5] Note that we focus explicitly on internet sales, not the use of the internet in a job more generally which could plausibly drive increased assortative matching.

across occupations, controlling for pre-internet trends and potentially confounding variables. Noting that our analysis should be interpreted as suggestive rather than definitive, we show that the results are consistent with the theoretical predictions of the model.

The paper proceeds as follows. Section 2 places our paper within the existing literature. Sections 3 and 4 present our theory of technological change. Section 5 brings some predictions of the model to the data. Section 6 provides concluding remarks.

2. The Literature

By modeling the scale of operation as the limit of IRS, this paper studies the effect of the facet of the ICT in that it increases this scale for certain occupations.[6] Typically, by modeling technology as a parameter of productivity, the economics literature focuses on the traditional technological progress that increases the quantity of output per unit of labour-time, whereas that facet of the ICT is not much studied. One exception is Garicano and Rossi-Hansberg (2014) who, building on Lucas Jr (1978), consider the implications of ICT innovation for the income distribution, which they model as a reduction in the rate at which the marginal return to the labor working for a manager falls. This reduction increases the size of firms in equilibrium, which can be regarded as analogous to the scale of operation for managers. This way of modeling ICT leads to different implications relative to our model. In particular, their approach has no effect on competition between managers, and no manager loses due to ICT innovation. In contrast, an increase in the IRS limit in our model increases competition between workers, and the lowest earners unambiguously lose. Related theoretical work on ICT includes Garicano and Rossi-Hansberg (2006, 2004), and Saint-Paul (2007) who examine the effects of reduced communication costs on the income distribution, where knowledge production and the organization of this production play an important role.

The general theme of our paper, namely that technological change may increase income inequality, fits within a large strand of literature that approaches the topic from a variety of perspectives. The dominant theoretical argument in this literature is the theory of Skill-Biased Technical Change (SBTC).[7] Relative to SBTC and its extensions,[8] our paper focuses on technological change that

[6]For a survey of research that studies the Internet from other angles see Levin (2011).

[7]Other theoretical studies on the effect of technology in general (not necessary the ICT) include Jones and Kim (2012) who endogenize the Pareto income distribution in a model in which technological progress augments the effects of entrepreneurs' efforts to increase productivity; and Saint-Paul (2006) who studies how productivity growth affects income inequality when consumers' utility from product variety is bounded from above.

[8]As just a few representative examples: Tinbergen (1974) is an example of early work linking the demand for skill to technology; Autor, Levy and Murnane (2003) find that computers displace routine workplace tasks and complement cognitive-intensive, non-routine tasks; Firpo, Fortin and Lemieux (2012) find an important role for technology in generating the observed inequality pattern over the 1980s, 1990s, and 2000s; Beaudry, Doms and Lewis (2010) find that computer adoption increases the return to skill; and Chen, Forster and Llena-Nozal (2013) find that technology has increased inequality across OECD countries. Recently, Acemoglu and Autor (2011) have extended the standard SBTC framework to endogenize the matching of skills to tasks.

differentially affects workers who are engaged in *the same type of work*.[9] As a result, our model can explain changes in within-occupation inequality, which SBTC has difficulty explaining. As we show in Section 5, within-occupation inequality in the U.S. has risen contemporaneously with the expansion of access to the internet. Furthermore, the occupations most exposed to the internet have seen relatively large increases in inequality. Thus, rising within-occupation inequality is important, and has been under-addressed in the literature. One exception is Helpman, Itskhoki, Muendler and Redding (2012) in the context of Brazil.

The role of scale of operation (or market access) as it relates to the return to skill has long been noted in the literature that seeks to explain certain features of the top of the income distribution, i.e., the earnings of "superstars"; for example, see Rosen (1981), Rosen (1983), Gabaix and Landier (2006), and Egger and Kreickemeier (2012), and see Neal and Rosen (2000) for a summary. However, this literature is mainly concerned with the level of income inequality for a *given* level of technology, and in particular with explaining how small differences in talent can lead to large differences in income. At the same time, it offers only an informal discussion regarding the potential impact of an increase in the scale of operation. In contrast, we model the scale of operation as the limit within which the production technology displays IRS, and this modeling approach is new to the literature. In addition, we also study selection into and out of occupations, i.e., the career choice margin, between providing labor and becoming a low-end professional (who are certainly not superstars), whereas this margin is absent in that literature.

Our model has some of the flavor of Melitz (2003),[10] though the two papers are concerned with different issues.[11] Both papers feature monopolistic competition with CES preferences, IRS, and agent heterogeneity. However, in our paper IRS operates up to some finite limit, whereas in Melitz (2003) this limit is infinity. As a result, while an increase in B in our paper might be regarded as parallel to an increase in the number of trading partners in Melitz (2003) (i.e., both reflect an increase in market size), the mechanism whereby this increase drives re-allocation is different. In Melitz (2003), it works through the factor market but it does not alter the price of any variety in the product market. In contrast, in our paper, the cost of the factor, which is labor, is unchanged, and the increased competition works through the product market, lowering the price of all varieties.

[9]The differential effects in our paper arise from differences in workers' human capital endowments, similar to the literature that is built on the matching model of Sattinger (1993). Also, in a recent paper Guvenen and Kuruscu (2012) show that simply adding heterogeneity in the ability to accumulate human capital to a standard SBTC framework generates a set of predictions for wages that matches features of the U.S. wage distribution over the past four decades in a simulation exercise, though there is no attempt to explain growth in within-occupation inequality.

[10]More accurately, our model is in line with Melitz (2003)-style models that incorporate heterogeneity in product quality, since the heterogeneity we introduce augments the marginal value of a unit of consumption, as in those models. For instance, see Baldwin and Harrigan (2007) or Kugler and Verhoogen (2012).

[11]Specifically, our paper is concerned with the effects of technological progress on the income distribution in a closed economy, while Melitz (2003) is focused on the relationship between exporting and aggregate productivity in an open economy.

On the other hand, this effect on the product market is captured by Melitz and Ottaviano (2008) using a model of monopolistic competition with quadratic preferences. However, this approach then leads to different implications for changes in market size. Whereas in Melitz and Ottaviano (2008) a larger market supports a greater number of varieties, in our paper an increase in B reduces the number of varieties, as an increased number of trading partners does in Melitz (2003).

3. The Model

The economy is populated by a continuum of agents. Agent $i \in [0, 1]$ is endowed with one unit of labor and h_i units of human capital. Without loss of generality, let h_i be increasing in i, that is $h_i' := \frac{dh_i}{di} \geq 0$. Agents choose to subsist on their labor endowment (i.e., muscles) or else on their human capital (i.e., brains).[12] In the latter case, they provide a stream of services which, to fix ideas, we assume throughout to be entertainment services (which we previously referred to more generally as professional services). The quality of the services provided by an agent depends on the size of her human capital endowment, as will be made clearer later.

Labor is used for producing both a subsistence good (such as food), which is used as numeraire, and entertainment services. The production of the subsistence good displays constant returns to scale. If L agents are employed to produce the subsistence good, then its aggregate output is

$$Y = AL.$$

Hence A represents the productivity of (unskilled) labor.

The production of entertainment services requires two factors, human capital and labor. We assume that human capital only affects the quality of the output and abstract from the effect of human capital on quantity. If an agent chooses to subsist on her human capital endowment and provide entertainment services, then her human capital impacts the quality of the entertainment services in two ways. First, some aspects of her human capital are unique, and thus so are the services that she provides (for instance, Jay-Z versus Madonna), as in the canonical Krugman (1979) model. As a result, each entertainer provides a unique variety of entertainment services, indexed by her identity $i \in [0, 1]$ and different agents' entertainment services compete under monopolistic competition. Secondly, the entertainment service provided with a higher level of human capital is of a better quality, in the sense that it gives consumers a higher value, as will be shown when we come to the utility of the agents. We abstract from the effect of human capital on output quantity by assuming that all agents have the same production function. Specifically, if an agent hires L

[12] Of course, in reality nearly all occupations require both muscle and brain. But clearly some occupations demand more human capital relative to labor, while others demand relatively more labor. For simplicity, we abstract from this continuum of human-capital-to-labor ratios, modeling it as a binary choice.

units of labor, the output of her variety is

$$y = \left\{ \begin{array}{c} \frac{A}{c}L, \text{ if } L \leq \frac{c}{A}B \\ B, \text{ if } L > \frac{c}{A}B \end{array} \right\}, \tag{1}$$

where $c > 0$ is a constant. Thus, if an agent decides to use his time to supply human capital rather than labor, thereby providing a variety of entertainment services, he can subsequently hire labor to produce output at constant returns to scale up to the limit B. According to this production function, a unit of labor produces A/c units of output, which means a unit of output needs c/A units of labor. Let w denote the wage of labor and F the opportunity cost of the agent's career choice (namely, time).[13] Then the cost function associated with producing entertainment services is

$$C(y) = \left\{ \begin{array}{c} F + w\frac{c}{A}y, \text{ if } y \leq B \\ \infty, \text{ if } y > B. \end{array} \right\}. \tag{2}$$

The marginal cost of production (up to B) is wc/A and stays constant. Due to the existence of fixed costs F, the average cost decreases with output y until $y > B$. Hence, if $B = \infty$, the production of entertainment services displays a typical instance of Increasing Returns to Scale (IRS). However, if $B < \infty$, then production of services displays IRS only up to the limit B, which is thus IRS up to some limit, which we denote as IRSL. The primary modeling innovation presented here is the introduction of this limit of IRS, denoted B. We use this B to represent the maximum scale of operation for an entertainer – for example, the capacity of the theater in which a musician performs.[14]

Agents have identical preferences. If an agent consumes s units of the subsistence good and e_i units of variety i of entertainment services, where $i \in E$ and E is the set of varieties of entertainment services available on the market, then her utility is[15]

$$\left(\mu s^{\widehat{\rho}} + \Big(\int_E (h_i e_i)^{\rho}\Big)^{\widehat{\rho}/\rho}\right)^{1/\widehat{\rho}}.$$

where $\mu > 0$ measures the relative importance of the subsistence good in the agent's utility function; $\widehat{\rho} < 1$ measures the substitutability or complementarity (as we allow $\widehat{\rho} < 0$) between the subsistence good and entertainment services; and $\rho \in (0,1)$ measures the substitutability between one entertainment service and another. We assume $\widehat{\rho} < \rho$, namely that the subsistence good is

[13] Since the alternative use of his time is to supply labor, we have that $F = w$.

[14] Alternatively, if the services that an agent provides consist of the management of a firm, then B represents the size of the firm (see footnote 9 above).

[15] Throughout the paper the notation "di" is omitted to simplify notation.

less substitutable for entertainment services than one variety of entertainment service is to another. Note that the marginal value of agent i's services is h_i, the same as the amount of her human capital. That is, entertainment services provided with higher human capital deliver greater value to consumers, as noted above.

Let p_i denote the price of variety i of entertainment services and let m_j denote the income of agent j. Then, the consumption decision that the agent faces is

$$\max_{s,\{e_i\}_{i\in E}} \left(\mu s^{\widehat{\rho}} + \Big(\int_E (h_i e_i)^{\rho}\Big)^{\widehat{\rho}/\rho}\right)^{1/\widehat{\rho}},$$
$$\text{s.t. } s + \int_E p_i e_i \leq m_j.$$

The agent's demand for the subsistence good and entertainment services are, respectively:[16]

$$s = m_j \cdot \frac{1}{1+\mu^{1/(\widehat{\rho}-1)} P^{\widehat{\rho}/(\widehat{\rho}-1)}} \tag{3}$$
$$e_i = m_j \cdot f(P,\mu) \cdot h_i^{\rho/(1-\rho)} p_i^{-1/(1-\rho)}, \tag{4}$$

where P is the general price of entertainment services per unit of quality, defined as

$$P := \left(\int_E (p_i/h_i)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}, \tag{5}$$

and

$$f(P,\mu) := \frac{P^{\frac{\rho-\widehat{\rho}}{(1-\rho)(1-\widehat{\rho})}}}{\mu^{\frac{1}{1-\widehat{\rho}}} + P^{\frac{\widehat{\rho}}{\widehat{\rho}-1}}}.$$

According to this optimal consumption plan, the demand for a variety is proportional to the agent's income. Hence, the aggregate demand for a particular variety that charges price p and is of quality h (equal to the human capital endowment of the provider) is

$$D(p;h) = M \cdot f(P,\mu) \cdot h^{\rho/(1-\rho)} p^{-1/(1-\rho)}, \tag{6}$$

where

$$M := \int_{[0,1]} m_j \tag{7}$$

is aggregate income. Note that $D'_h > 0$ – that is, given some price, the demand for a higher-quality variety is greater because consumers derive greater value from it.

[16] In the model each professional sells to all agents and each agent buys from all professionals. This is a result of the fact that in the model consumers are homogenous, with identical utility functions. In reality, no musician sells to the entire population (with the possible exception of Michael Jackson in 1982) and no one buys music from all musicians. But if we aggregate the consumption of all music and imagine that it is consumed by one "representative agent", then the model makes sense in terms of tracking aggregate demand for each musician.

If an agent with human capital h chooses to supply labor and produce the subsistence good, she gets A. Hence in equilibrium the wage of labor employed in the production of entertainment services is also A, that is, $w = A$. Therefore, by (2), the marginal cost of producing entertainment services up to scale B is $w\frac{c}{A} = c$, which is independent of A. If the agent chooses to live on her human capital and produce her variety of services, the demand for her services will be given by (6), where she takes the aggregate variables P and M as given. She then sets the price of her services by solving the following decision problem:

$$m(h) = \max_p (p - c)D(p; h), \text{ s.t. } D(p; h) \leq B \tag{8}$$

An agent with human capital h chooses to provide entertainment services instead of supplying labor only if

$$m(h) \geq A \tag{9}$$

From the envelope theorem and (8), $m'(h) > 0$. There thus exists a threshold $k \in [0, 1]$ such that agent i chooses to provide entertainment services if and only if $i \geq k$, where k is pinned down by

$$m(h_k) = A.^{17} \tag{10}$$

If $i < k$, agent i earns wage $w = A$, and if $i \geq k$ agent i earns $m(h_i)$, the rents associated with her human capital. Hence the set of available entertainment services is $E = [k, 1]$. It follows that the general price for entertainment services, from (5), is given by

$$P = \Big(\int_k^1 (p_i/h_i)^{\rho/(\rho-1)} \Big)^{(\rho-1)/\rho}, \tag{11}$$

and aggregate income is

$$M{=}kA + \int_k^1 m(h_i). \tag{12}$$

Definition 1. *A profile (P, k, M) forms a competitive equilibrium if*

(i) P is given by (11), where p_i solves (8) with $h = h_i$;

(ii) agent i chooses to supply labor if and only if $i < k$ where k is determined by (10);

[17] More generally, k satisfies $\left\{ \begin{array}{c} k = 0 \text{ if } m(h_0) > A \\ k = 1 \text{ if } m(h_1) < A \\ m(h_k) = A \text{ if } m(h_0) < A < m(h_1) \end{array} \right\}$. The first two cases capture the possibilities that no one produces the subsistence good and that no one produces any entertainment services. With CES preferences, neither occurs in equilibrium because if no one produces the subsistence good, the marginal utility from consumption of it will be infinitely large, and providing it will be very profitable. This argument also applies to the case in which no one provides entertainment services.

(iii) Aggregate income is given by (12).[18]

4. Equilibrium and Technological Change

In this section we prove the existence of a unique equilibrium, find the equilibrium income of each agent, and then consider comparative statics with respect to B. In the final subsection we also consider the effect of an increase in A on the income distribution. We first focus on the case in which the capacity constraint, $D(p;h) \leq B$, is binding for all agents who choose to be entertainers, such that their profit is $(p-c)B$. This effectively requires B to be sufficiently small and an exact condition is provided in Subsection 4.3. In that subsection, we also show that the insights derived from this case can then be applied straightforwardly to the case in which the capacity constraint is not binding for some subset of entertainers. Of course, if it is not binding for any agents then an increase in B will have no effect.

To find the equilibrium income distribution, we proceed in two steps. First, we find the income of each agent, given that the identity of the marginal entertainer is k. Second, we find an equation that determines k and show that the equation has a unique root for k.

Two observations immediately follow from the fact that the capacity constraint is binding for each entertainer. First, the supply of each variety of services is the same, B. As a result, each agent, whatever his income, consumes the same quantity of each variety; put differently, his consumption of entertainment services is a multiple of a bundle that consists of one unit of each variety. To see this, observe that according to (4) any agent's consumption of variety i is proportional to his income and this proportion is the same across agents, denoted by x_i. Our claim is equivalent to $x_i = x_j$ for any two varieties i and j. To see that this is true, note that the aggregate consumption of variety i is Mx_i and equals the aggregate supply, B. It follows that $x_i = B/M$ for any variety i.

Second, with $D(p;h)$ given by (6), the binding capacity constraint, $D(p;h) = B$, implies that the price of variety i is:

$$p_i = \Big(\frac{Mf(P,\mu)}{B}\Big)^{1-\rho} h_i^{\rho}. \tag{13}$$

Thus, an agent with higher human capital charges a higher price for her services because they deliver a higher value to consumers. In fact, the price is proportional to the marginal value raised to power $\rho < 1$ – that is, h_i^{ρ} – because one variety of entertainment services is not a perfect substitute for another, in general. In the special case in which it is – that is $\rho = 1$ – the price of a variety is then directly proportional to its marginal value. It follows that the aggregate spending on variety

[18] We skip the clearing of the subsistence good market, which pins down the fraction of labor used for producing the good, a variable that is not very interesting in the context of this paper.

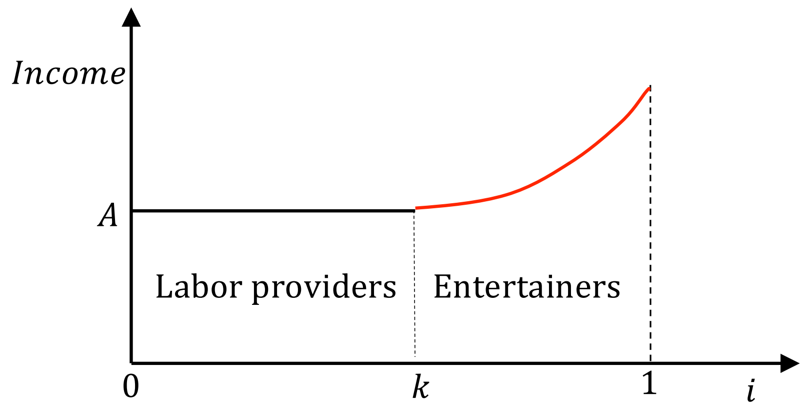

Figure 1: The Equilibrium Income Distribution

i, namely $p_i B$, is a fraction of h_i^ρ / H_k^ρ of the aggregate spending on all the entertainment services $\int_k^1 p_j B$, where

$$H_k := \left\{ \int_k^1 h_i^\rho \right\}^{\frac{1}{\rho}} \tag{14}$$

is the aggregate quality of the aforementioned bundle which consists of one unit of each variety.[19] Moreover, for any $i \geq k$, $p_i/p_k = h_i^\rho / h_k^\rho$. Agent k, the marginal entertainer, obtains profit A as she is indifferent between the two occupational choices. That is, $p_k \times B - Bc = A$, or

$$p_k = A/B + c. \tag{15}$$

It follows that $p_i = (A/B + c) \times h_i^\rho / h_k^\rho$ and the income of agents $i \geq k$ is $m_i = (p_i - c)B = (A + Bc)h_i^\rho / h_k^\rho - Bc$. We know that agents $i < k$ choose to provide labor and earn $m_i = A$. Putting these together, the equilibrium income distribution is:

$$m_i = \left\{ \begin{array}{l} A \text{ if } i < k \\ (Bc + A)\frac{h_i^\rho}{h_k^\rho} - Bc \text{ if } i \geq k \end{array} \right\} \tag{16}$$

This income distribution is illustrated in Figure 1.[20]

We now move on to determining k via the market clearing condition for the subsistence good. We

[19] In this aggregation, as in (13), the quality of each variety is raised to power $\rho < 1$ because one variety of entertainment services is not a perfect substitute for another.

[20] The figure is based on the assumption that h_i is a convex function of i so that m_i, though a concave function of h_i, is convex in i. Roughly, the assumption is that within a typical talent distribution, there are a small number of people at the top who are much more talented than the rest – a view that seems consistent with the evidence.

first establish that aggregate spending on this good is $BH_k(\mu P)^{\frac{1}{1-\widehat{\rho}}}$. To see this, note that from (3) it follows that the aggregate income spent on the subsistence good is $M \times \left[1+\mu^{1/(\widehat{\rho}-1)}P^{\widehat{\rho}/(\widehat{\rho}-1)}\right]^{-1}$. To find the aggregate income M, observe that with the price of each variety given by (13), the price index, from (11), is

$$P = \Big(\frac{Mf(P,\mu)}{B}\Big)^{1-\rho} H_k^{\rho-1}. \tag{17}$$

With $f(P,\mu) = P^{\frac{\rho-\widehat{\rho}}{(1-\rho)(1-\widehat{\rho})}} / \left(\mu^{\frac{1}{1-\widehat{\rho}}} + P^{\frac{\widehat{\rho}}{\widehat{\rho}-1}}\right)$, it follows that

$$M = BPH_k \times \left[1+\mu^{\frac{1}{1-\widehat{\rho}}}P^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}\right]. \tag{18}$$

Therefore, aggregate spending on the subsistence good is

$$BPH_k \times \left[1+\mu^{\frac{1}{1-\widehat{\rho}}}P^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}\right] \times \left[1+\mu^{1/(\widehat{\rho}-1)}P^{\widehat{\rho}/(\widehat{\rho}-1)}\right]^{-1} = BH_k(\mu P)^{\frac{1}{1-\widehat{\rho}}}$$

Next, we find the aggregate supply of the subsistence good. A mass $1-k$ of agents provide services, each demanding Bc/A units of labor as input. The total labor supply is k. Thus, $k - \frac{c}{A}B \times (1-k)$ agents work to produce the subsistence good, yielding an output of $[k - \frac{c}{A}B \times (1-k)] \times A = kA - (1-k)cB$. Note that this aggregate supply of the subsistence good can be re-written as $(A+Bc)(k-k_0)$, where $k_0 = \frac{Bc}{A+Bc}$ is the threshold for the number of agents at which the aggregate supply of the subsistence good is zero.

Market clearing for the subsistence good thus implies:

$$BH_k(\mu P)^{\frac{1}{1-\widehat{\rho}}} = (A+Bc)(k-k_0). \tag{19}$$

This equation contains P, an endogenous variable. To determine k, we exploit an additional connection between P and k, as follows. Using (17) to cancel $\left(\frac{Mf(P,\mu)}{B}\right)^{1-\rho}$ in (13), we find $p_i = PH_k^{1-\rho}h_i^{\rho}$ for any $i \geq k$, in particular when $i = k$. At the same time, in (15) we found $p_k = A/B + c$. Therefore,

$$PH_k^{1-\rho}h_k^{\rho} = \frac{A}{B} + c. \tag{20}$$

Solving for P, substituting it into (19) and rearranging, we arrive at a single equation that pins down k in equilibrium:

$$\mu^{\frac{1}{1-\widehat{\rho}}} H_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}} h_k^{\frac{-\rho}{1-\widehat{\rho}}} \times (A/B+c)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}} = k - k_0 \tag{21}$$

As noted above, the term on the right hand side of this equation is the aggregate supply of the subsistence good, measured in units of the revenue of the marginal entertainer (i.e., $A + Bc$), if his identity is k. The term on the left hand side of this equation, as we have explained above, is aggregate spending on the subsistence good conditional on the marginal entertainer being agent k, also measured in units of his revenue. This can be clearly seen for the Cobb-Douglas case, where $\widehat{\rho} = 0$. In this case, this term simplifies as $\mu H_k^\rho / h_k^\rho$. Measured in units of the agent's revenue, the spending on his service (the agent's revenue) is 1. Since this spending is h_k^ρ / H_k^ρ of the aggregate spending on entertainment services, the aggregate spending on entertainment services is therefore H_k^ρ / h_k^ρ, and then μ times this term gives the aggregate spending on the subsistence good in the Cobb-Douglas case, where the ratio of the spending on the subsistence good to that on entertainment services is always μ, independent of the price index, P.[21]

For the non-Cobb-Douglas case, this ratio depends on the prices of services and hence we have the price adjustment term $(A/B + c)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}} = (p_k)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}$. If the prices of entertainment services change, this generates the two standard, and conflicting, effects on the demand for the subsistence good, namely the substitution and income effects. The price adjustment term represents the net of these two effects. In particular, suppose $\widehat{\rho} > 0$. Then when entertainment services are cheaper, reflected in a smaller p_k, spending on the subsistence good will be reduced, because in this case the substitution effect dominates the income effect.

The aggregate supply of the subsistence good – the right hand side term – increases with k. At the same time, aggregate spending – on the left hand side – decreases with k, the identity of the marginal entertainer, and spending goes to zero as k goes to 1.[22] Intuitively, this is because an agent with relatively little human capital chooses to become an entertainer only if the economy is rich enough such that a large enough aggregate income is spent on entertainment services. Put differently, if the marginal entertainer has a relatively high human capital level – i.e., k is big – then the economy must be poorer, which means the aggregate spending on the subsistence good is smaller too. In the extreme case, if the economy can support only the agent with the greatest human capital as an entertainer – i.e., $k = 1$ – then it must be extremely poor in that aggregate income approaches zero and each agent can only spare a tiny amount of income to spend on entertainment services.

Equation (21) can be re-arranged into

$$\mu^{\frac{1}{1-\widehat{\rho}}} H_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}} h_k^{\frac{-\rho}{1-\widehat{\rho}}} = (A/B + c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}} (k - k_0), \tag{22}$$

[21] In the Cobb-Douglas case, each agent spends a fraction of $\frac{\mu}{1+\mu}$ of his income on the subsistence good and that of $\frac{1}{1+\mu}$ on entertainment services. Hence the aggregate spending on the former good is μ times that on the latter good.

[22] Since $\rho - \widehat{\rho} > 0$, $H_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}}$ increases with $H_k = \{\int_k^1 h_i^\rho\}^{\frac{1}{\rho}}$, which decreases with k. Since $\rho > 0$, $h_k^{\frac{-\rho}{1-\widehat{\rho}}}$ decreases with h_k which, by assumption, increases with k. Moreover, $H_1 = 0$. Hence the term on the left hand side equals 0 at $k = 1$.

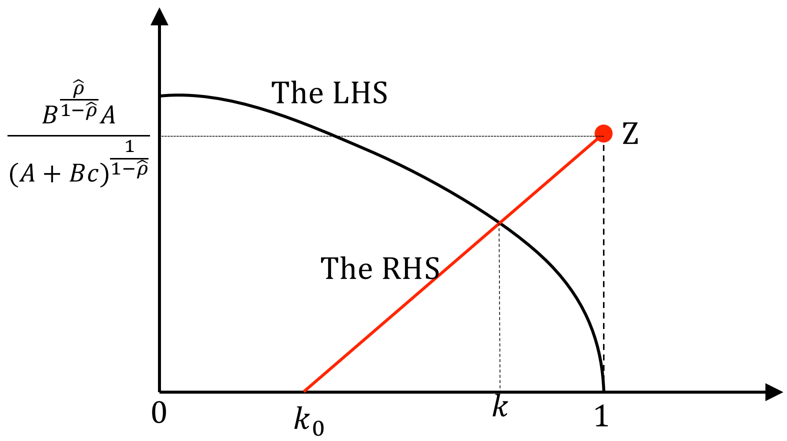

Figure 2: The Existence and Uniqueness of Equilibrium

As argued above, the left hand side of (22) decreases from a positive number to 0 with k ascending from k_0 to 1, and with this movement of k the right hand side of (22) linearly increases from 0 to some positive number. Both sides are depicted in Figure 2. This argument leads to the following proposition:

Proposition 1. *A unique equilibrium exists, in which* $k \in (k_0, 1)$, *and is given by (22).*

4.1. Technological Changes that Expand the Limit of IRS

Here we consider the comparative statics with respect to B, the IRS limit.[23] We consider first how an increase in B affects the occupational choice of the agents, captured by k, and then consider its effect on the income distribution. The equilibrium k is determined by equation (21), which is derived from the market clearing condition associated with the subsistence good. From this equation, we can see that an increase in B generates two effects that impact k. First, the term on the right hand side of (21), which represents the aggregate supply of the subsistence good, goes down because $k_0 = Bc/(A + Bc)$ increases with B. Intuitively, a larger B means that more labor is required as input into the production of entertainment services. Hence, given the total supply of labor (i.e., k), fewer workers are employed to produce the subsistence good and less of the good is produced. As result, given the demand for the subsistence good, a rise in B implies that more agents will supply labor in order to meet the demand. That is, the supply-side effect alone drives k up.

[23]Since we are examining the case in which the capacity constraint, $D(p; h) \leq B$, is binding, the comparative statics are based on the assumption that it remains binding following the change we are considering. Later we consider the comparative statics for the case in which the capacity constraint is binding for some share of entertainers.

The second effect, from the left hand side of (21) (which represents the aggregate demand for the subsistence good), is that the price adjustment term, $(A/B+c)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}$, changes with B. If $\widehat{\rho}<0$, this term increases with B, and so does the aggregate demand for the subsistence good. Intuitively, when the quantity of each variety increases, entertainment services in general become relatively cheaper. This increases the demand for the subsistence good if $\widehat{\rho}<0$ because in this case the income effect dominates the substitution effect. A greater demand for the subsistence good induces more agents to provide labor – that is, it also pushes k up. Therefore, if $\widehat{\rho}<0$, the effect on the demand side moves k in the same direction as that on the supply side, namely upwards.

However, if $\widehat{\rho}>0$, the price adjustment term $(A/B+c)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}$ decreases with B, and so does the demand for the subsistence good, due to the fact that the substitution effect dominates the income effect in this case ($\widehat{\rho}>0$). In this case, the demand-side effect alone drives k down, in the opposite direction to the supply-side effect. The trade-off between them determines its net movement – up or down. As the demand-side effect vanishes at $\widehat{\rho}=0$, where the substitution effect is exactly offset by the income effect, by continuity, we expect this effect to be weak and dominated by the positive effect on the supply side – hence k to go up – if $\widehat{\rho}$ is close enough to zero. One such condition is given in the following proposition.

Proposition 2. *If*

$$\widehat{\rho} \le \frac{Bc}{A+Bc}, \tag{23}$$

then $dk/dB>0$. That is, with an increase in the limit of IRS, fewer agents choose to provide entertainment services and the number of varieties provided falls.

Proof. We relegate the proof to Appendix A.1. ∎

To explain the intuition for condition (23), we turn to equation (22), the two sides of which are pictured in Figure 2. We note that the left hand side is independent of B. As a result, the curve in Figure 2, representing the LHS, is invariant to an increase in B. The RHS, on the other hand, is affected in two ways. First, $k_0=Bc/(A+Bc)$ increases with B so that k_0 moves rightward, from k to k'. Second, the uppermost part of the line, Z, may shift up or down. If Z moves down then k shifts further to the right (to the position k''), as is illustrated in the right panel of the Figure.

In addition, the height of Z is $AB^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}/(A+Bc)^{\frac{1}{1-\widehat{\rho}}}$. Thus, Z moves down with an increase in B if $d\left[AB^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}/(A+Bc)^{\frac{1}{1-\widehat{\rho}}}\right]/dB\le 0$, which is equivalent to (23). This condition is assumed to hold in the rest of this subsection.

According to Proposition 2, if $\widehat{\rho}$ is small then an increase in B leads the marginal entertainer to be driven out of the entertainment occupation and to thus become a labor provider.

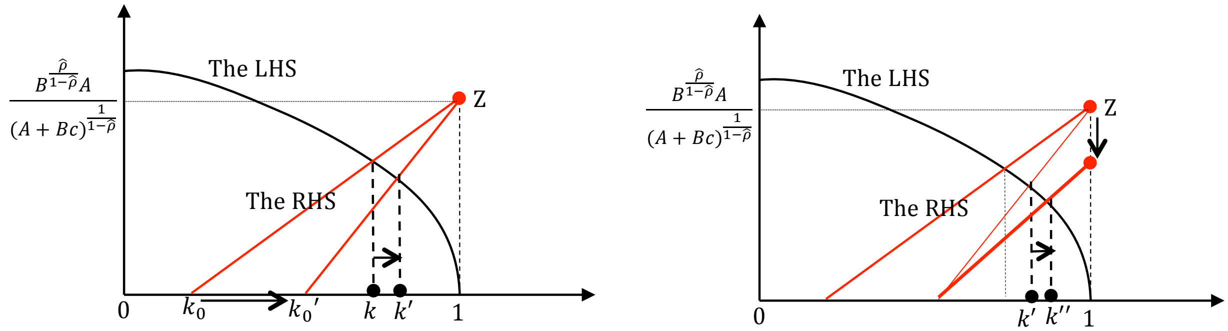

Figure 3: The effect of an increase in B on k. The left panel: an increase in B moves k_0 to k', which increases k to k'. The right panel: if point Z moves down, then k is shifts further to k''

Note that in our model economy, agent i's net gain from becoming a professional relative to providing labor is $(p_i - c) \times B - A$. If an increase in B pushes the agent out of the profession – that is, if this net gain becomes negative – then this must be due to a decrease in p_i – that is, the price of her variety is reduced due to heightened competition. This is a difference with Melitz (2003) where the price of each variety does not respond to changes in the number, or prices, of competing varieties. Indeed, in our setting, in which we consider IRSL rather than IRS, all the varieties get cheaper with an increase in B, as stated in the following lemma:

Lemma 1. *For $i > k$, $dp_i/dB < 0$.*

Proof. From the discussion sitting between equations (14) and (15), we know that $p_i = p_k \times h_i^\rho / h_k^\rho$ and $p_k = A/B + c$. Thus $d \log p_i/dB = d \log p_k/dB - d \log h_k^\rho/dB < -d \log h_k^\rho/dB = -(\rho h_k'/h_k \times dk/dB) < 0$, as $h_k' > 0$ and $dk/dB > 0$ by Proposition 2. ∎

Having examined the effect of an increase in B on occupation choice, we move on to considering its effects on the income distribution. Two effects are presented below. The first is a direct implication of Proposition 2: fiercer competition due to an increase in B generates losses for lower-end entertainers. Consider those entertainers endowed with a level of human capital close to the marginal entertainer's, and who are therefore squeezed out of the entertainment business with the increase in B. Before the rise in B they earned strictly more than the wage of labor, A, as they strictly preferred being an entertainer to providing labor. After the increase in B they are squeezed out and subsequently provide labor, and therefore earn the wage of labor, A. These agents therefore lose. This result is stated as the following proposition:

Proposition 3. *There exists $\widehat{k} > k$ such that $dm_i/dB < 0$ for $i \in (k, \widehat{k})$ – namely, the lower-end entertainers lose from an increase in the limit of IRS.*

Proof. We relegate the proof to Appendix A.2. ∎

The second effect of a rise in B for the income distribution is that it increases income equality within the entertainment occupation, as the following proposition states:

Proposition 4. *For $i > k$, $\frac{d \log m_i}{dB}$ strictly increases with m_i – namely the rate of change in the income of entertainers is positively correlated with their present income.*

Proof. We relegate the proof to Appendix A.3. ∎

According to this proposition, the higher the current income of an entertainer, the more the entertainer gains (or the less she loses) following an increase in the limit of IRS. This leads to growth in income inequality within the entertainment occupation. The intuition for the proposition is as follows. An increase in B impacts the entertainers' revenues in two ways (while also increasing their cost, Bc). First, a positive effect: a rise in B expands entertainers' capacity and thereby increases their revenues. Second, a negative effect: since all entertainers are equally exposed to the increased capacity, the competition between them becomes fiercer, resulting in lower prices of entertainment services which reduces revenues (all else equal). Whereas all entertainers face the same degree of competition, an entertainer who has relatively more human capital – and thus earns relatively more – receives a greater gain from the enlargement in capacity because she provides a better quality of service and is therefore able to charge a higher price for her variety. As a result, entertainers with initially higher earnings gain more, or lose less, from an increases in B. Indeed, Proposition 3 shows that if an entertainer's human capital is low enough, then his gains from the enlargement of capacity is dominated by the losses due to fiercer competition.

We note that a corollary of Proposition 4 is that if $i > j > k$ and thus $m_i > m_j$, then $d \log m_i/dB > d \log m_j/dB$. That is, the log wage gap between agents i and j increases with B. If we let i and j be an agent respectively at the 90th and 10th (or 50th) percentile of the income distribution, then a corollary of Proposition 4 is that $\log w_{90\%} - \log w_{10\%}$ (or $\log w_{90\%} - \log w_{50\%}$) grows due to an increase in the scope of operation for the occupation. This corollary provides the theoretical foundation for our later regression specification given by equation (28).

As noted previously, the gain to an entertainer from the enlargement of capacity is proportional to her human capital endowment raised to the power ρ (as is the price she charges). If an entertainer's human capital endowment is high enough the increase in revenue will outweigh the losses due to fiercer competition, and the entertainer will reap a net gain due to the increase in B. To state this formally, let

$$\Omega(\rho) := \max_{x \in [k_0, 1]} \frac{\rho \cdot h'(x)/h(x)}{1 + \rho \cdot h'(x)/h(x) \cdot (x - k_0)}.$$

We can then state the following:

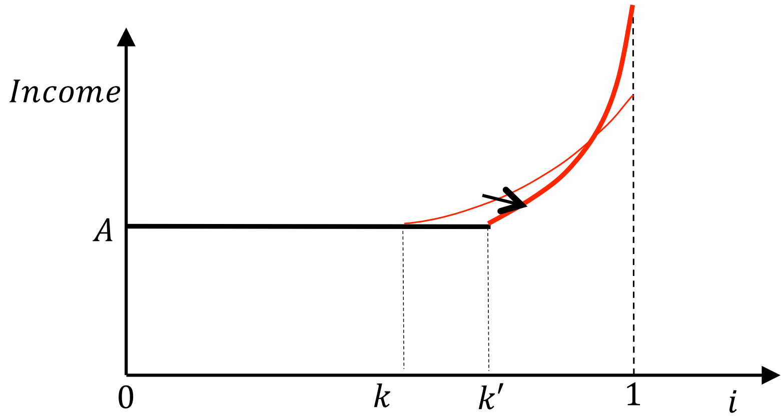

Figure 4: An increase in B squeezes the lower-end entertainers out, and raises income inequality within the entertainment occupation.

Lemma 2. *Assume that* $\Omega(\rho) \cdot A/(A + Bc) < 1$ *and* $\widehat{\rho} \geq 0$*. Then* $dm_i/dB > 0$*, namely agent* i*'s income rises with an increase in the limit of IRS as long as*

$$\frac{h_i}{h_k} > \left(\frac{1}{1 - \Omega(\rho) \cdot A/(A + Bc)} \right)^{\frac{1}{\rho}}. \tag{24}$$

Proof. We relegate the proof to Appendix A.4. ∎

Condition (24), however, is not easy to check. This is because k is determined in equilibrium and depends on the distribution of human capital (specifically, the functional form of $h(i)$). We therefore present an approach, dispensing with k, to obtain a condition under which the top entertainers gain on net from an increase in the limit of IRS.

Let $f(k_0, y)$ denote the unique solution for $t \in [k_0, 1]$ in

$$t - k_0 = y(1 - t)^{\frac{\rho - \widehat{\rho}}{\rho(1 - \widehat{\rho})}},$$

and let

$$D := \mu^{\frac{1}{1 - \widehat{\rho}}} (A/B + c)^{\frac{\widehat{\rho}}{1 - \widehat{\rho}}}.$$

Lemma 3. *Assume* $h_1 > 1$*. If for some* ζ*,* $h_1 \geq \zeta \cdot h(f(k_0, D \cdot \zeta^{\frac{\rho}{1 - \widehat{\rho}}}))$*, then* $h_1 > \zeta \cdot h_k$*.*

Proof. We relegate the proof to Appendix A.5. ∎

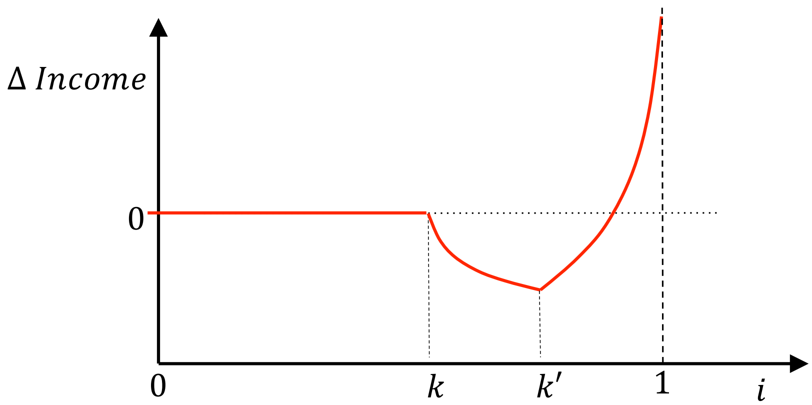

Figure 5: Income growth due to an expansion in B.

The two lemmas above lead to the following proposition, which gives a condition for the distribution function of human capital under which the top entertainers' income strictly increases with B. Let

$$\xi := [\frac{1}{1-\Omega(\rho)\cdot A/(A+Bc)}]^{\frac{1}{\rho}}.$$

Proposition 5. *Assume that* $\Omega(\rho)\cdot A/(A+Bc) < 1$ *and* $\widehat{\rho} \geq 0$. *If* $h_1 > 1$ *and* $h_1 \geq \xi \cdot h(f(k_0, D\cdot \xi^{\frac{\rho}{1-\widehat{\rho}}}))$, *then* $dm_1/dB > 0$.

Thus, if agents have enough human capital then they reap a net gain from an increase in the limit of IRS. Note that the condition in this proposition focuses on only two points of the function $h(i)$, namely at $i = 1$ and $i = f(k_0, D\cdot\xi^{\frac{\rho}{1-\widehat{\rho}}})$, and hence it can be satisfied by any distribution of human capital in which $h(1)$ is sufficiently large. When this proposition holds, the top entertainers gain on net from an increase in the limit of IRS. This result, together with Proposition 2 which states that entertainers at the bottom of the distribution are pushed out of the entertainment occupation into providing unskilled labor, implies that an increase in B leads to a change in the income distribution depicted in Figure 4. This results in a U-shaped change in income across agents, as illustrated in Figure 5.

4.2. An Increase in the Productivity of Unskilled Labor

We now consider the comparative statics with respect to A, the productivity of (unskilled) labor. As in the case of an expansion in B, we first explore how an increase in A affects agents' occupational choices and, second, we explore its effect on the income distribution. To begin, observe that a rise in

A directly increases the income of labor, but has no direct impact on the income of entertainers. That is because although an increase in labor productivity increases the productivity of entertainment services since fewer workers are needed to produce the same amount of entertainment, it meanwhile increases the wage of labor and, on net, the marginal (labor) cost of producing entertainment services, $w \times c/A$, stays constant at c, since $w = A$. In this sense we say that an increase in A is biased toward (unskilled) labor. It seems at first glance that an increase in A would induce more agents to provide labor, fewer to become entertainers, and would reduce income inequality. However, we show below that these direct effects are in fact fully offset by a general equilibrium effect. Specifically, an increase in A raises aggregate income, which will raise the spending on entertainment services and consequently enrich entertainers. This general equilibrium effect, we will show, dominates the direct effect for the impact on both occupation choice and the income distribution.

To consider the impact on occupation choice, we return to equation (21), which determines equilibrium k via market clearing of the subsistence good. The supply side (the right hand side of the equation) increases with A since $k_0 = Bc/(A + Bc)$ decreases with it. Intuitively, given the quantity of labor k, output of the subsistence good rises with labor productivity. For fixed demand, this effect on the supply side induces fewer agents to produce the subsistence good – that is it induces k to go down. On the demand side (the left hand side of equation (21)), the movement depends on the price adjustment factor, $(A/B+c)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}$. This factor decreases with A if $\widehat{\rho} \leq 0$. Intuitively, a larger A induces a greater supply of the subsistence good, which therefore makes entertainment services relatively more expensive – as $p_k = A/B + c$ increases with A. This change induces a substitution effect and an income effect with respect to the demand for the subsistence good. If $\widehat{\rho} \leq 0$, the latter dominates the former, on net reducing the demand for the subsistence good. It follows that the force on the demand side, similar to that on the supply side, drives k down. Hence, if $\widehat{\rho} \leq 0$, $dk/dA < 0$.

However, the price adjustment factor, $(A/B + c)^{\frac{\widehat{\rho}}{1-\widehat{\rho}}}$ increases with A if $\widehat{\rho} > 0$, in which case the substitution effect dominates the income effect and hence a rise in the relative price of entertainment service increases demand for the subsistence good. This effect on the demand side alone, then, induces k to go up. The effect, however, vanishes if $\widehat{\rho} = 0$ in which case the substitution effect is exactly offset by the income effect. By an argument of continuity, we expect that this demand-side effect will be dominated by the negative effect on the supply side when $\widehat{\rho}$ is small. Hence,

Proposition 6. *If $\widehat{\rho} \leq \frac{Bc}{A+Bc}$ – i.e., (23) holds – then $dk/dA < 0$.*

Proof. We relegate the proof to Appendix A.6. ∎

To provide intuition for this condition, we again highlight equation (22), the two sides of which are depicted in Figure 6. The LHS, represented by the curve, is independent of A. Thus, the curve in Figure 6 does not shift with an increase in A. As for the RHS, an increase in A shifts the straight line in Figure 6 in two ways. First, $k_0 = Bc/(A + Bc)$ falls with an increase in A and the position of k_0 shifts leftward to the position of k_0'. Second, the uppermost part of the line, Z, may move up or down. If Z moves upward, then k falls further to k'', as illustrated by the right panel of the Figure.

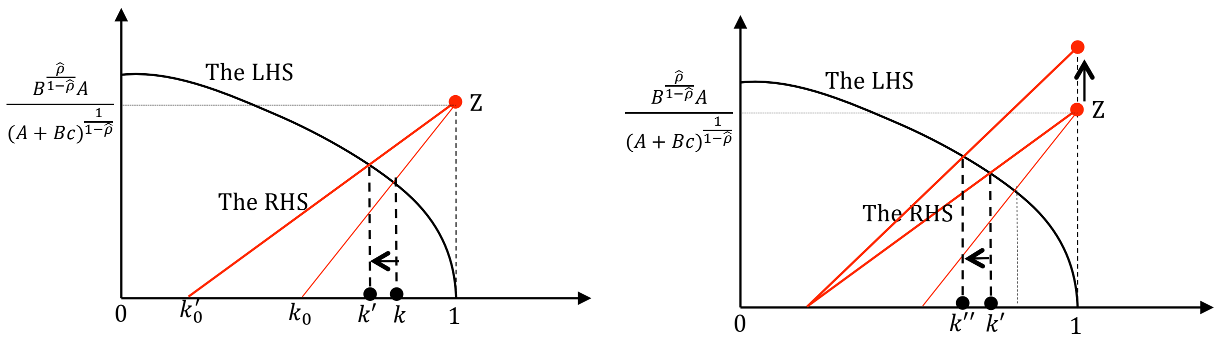

Figure 6: The effect of an increase in A on k. The left panel: an increase in A moves k_0 to the left, which decreases k to k'. The right panel: if Z moves upward, then k falls further to k''

The height of Z is $AB^{\frac{\hat{\rho}}{1-\hat{\rho}}}/(A+Bc)^{\frac{1}{1-\hat{\rho}}}$. Z moves upward with an increase in A if

$$\frac{dAB^{\frac{\hat{\rho}}{1-\hat{\rho}}}/(A+Bc)^{\frac{1}{1-\hat{\rho}}}}{dA} \geq 0,$$

which is equivalent to (23).

Thus, with a rise in labor productivity more agents choose to provide entertainment services, and the number of varieties therefore increases. This means that the general equilibrium effect dominates the direct effect with respect to the impact of an increase in A on occupation choice.

Having examined the effect of an increase in A on occupation choice, we move on to considering its effects on the income distribution. We noted that an increase in A directly benefits unskilled labor, but has no direct impact on the income of entertainers. However, we have also noted that entertainers will gain indirectly from the general equilibrium effect. In fact, they gain more than the labor providers according to the following proposition – and the more they currently earn, the more they gain. Hence, the general equilibrium effect dominates the direct effect here as well. To understand this proposition, note that if agent j provides labor, then his income is $m_j = A$ and hence $dm_j/dA = 1$.

Proposition 7. *If $i \geq k$, namely if agent i is an entertainer, then $\frac{dm_i}{dA} > 1$. Moreover, $\frac{dm_i}{dA}$ is positively correlated with m_i.*

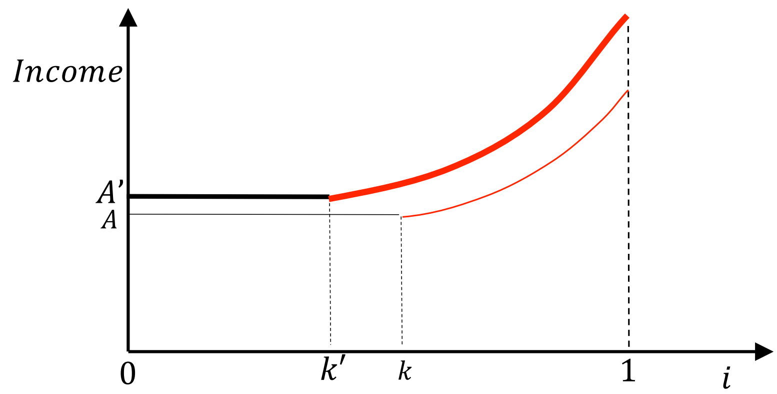

Figure 7: An increase in A raises all agents' incomes while also increasing income inequality.

Proof. We relegate the proof to Appendix A.7. ∎

Of the two results presented in the proposition, the second one can be intuitively explained as follows. The major effect of an increase in labor productivity is to raise aggregate income. With the economy becoming richer, the agents spend more on entertainment services. As an entertainer, agent i acquires a fraction h_i^ρ / H_k^ρ of aggregate spending on entertainment services. Hence, the more an entertainer currently earns, i.e., the higher is her human capital, the greater is the growth in her income from an increase in A.

To understand the first result – namely, that all entertainers gain more than all labor providers due to an increase in A – we only need to understand why the marginal entertainer gains more than any of the labor providers. This is a direct implication of the change in occupation choice, as stated in Proposition 6. The marginal entertainer earns $m_k = A$. If her earnings rise with A less than one to one, she would strictly prefer to provide labor with the increase in A and then k would increase, which is not the case by Proposition 6. Hence, the marginal entertainer gains more than any labor provider due to an increase in A.

By Proposition 7, for the whole population, the more an agent currently earns, the more he gains from an increase in A. Therefore, *a rise in the productivity of unskilled labor increases overall income inequality.* The effect on the income distribution is illustrated in Figure 7.

4.3. Discussion: When the Capacity Constraint Is Non-Binding for Some Entertainers

Thus far we have considered the case in which the capacity constraint, $D(p; h) \leq B$, is binding for all entertainers. If the capacity constraint is non-binding for some entertainers, then these entertainers' human capital will lie at the lower end of the distribution. The demand for an entertainer's services, by (6), is proportional to $h_i^{\rho/(1-\rho)}$. Thus, the profit-maximizing output in the absence of the capacity constraint increases with h_i. As a result, if it is binding for agent i then it is binding for all the agents $i' \geq i$, and if it is not binding for agent i, then neither is it for any agent $i' \leq i$. Thus, if and only if the capacity constraint is binding for the marginal agent k, will it be binding for all entertainers. Since the entertainers' problem is given by (8), in the absence of a capacity constraint, the optimal price is c/ρ. The constraint is binding for agent k if he cannot reach this price by supplying enough output, namely if the price pinned down by the binding capacity constraint, p_k, is above c/ρ. This condition, with p_k given by (13) with $i = k$, formally is:

$$\left(\frac{Mf(P,\mu)}{B}\right)^{1-\rho} h_k^{\rho} > \frac{c}{\rho}. \tag{25}$$

Hence if this condition holds, then the relevant equilibrium is the case in which the capacity constraint is binding for all the entertainers, and the previous analysis holds.

If the condition does not hold, then the capacity constraint is binding for some share of entertainers and non-binding for the remainder. The argument above implies that there exists $i^* \in (k, 1)$ such that it is non-binding for $i < i^*$ and binding for $i > i^*$. In particular, it is non-binding for the marginal entertainer, k. In this case, the propositions derived above all hold true qualitatively. Formally:

1. Proposition 1 still holds. The unique equilibrium still exists, and is driven by the same economic forces as before. If too many agents choose to provide labor and produce the subsistence good, then the entertainment services will be expensive, which will induce further entry. Conversely, if too few agents provide labor there will be entry into production of the subsistence good.
2. Proposition 2 still holds, and therefore so does Proposition 3 which is a consequence of Proposition 2; that is, an increase in B squeezes the entertainers at the lower end out of the profession (i.e., $dk/dB > 0$). In fact, this holds even under a condition less strict than (23). That is because the marginal entertainer, now with a non-binding capacity constraint with a present value of B, gains nothing from an increase in B, whereas in the case of his capacity constraint being binding, he obtains a positive effect due to the loosening of the constraint. In the absence of this offsetting positive effect, the marginal entertainer is pushed out of the profession by an even stronger force.
3. Proposition 4 still holds, and is formalized in the following proposition.

Proposition 8. *Suppose there exists $i^* \in (k, 1)$ such that the capacity constraint is non-binding for $i < i^*$ and strictly binding for $i > i^*$. Then for $i > k$, $\frac{d \log m_i}{dB}$ increases with m_i*

and this increase is strict for $i > i^*$. See Appendix A.8 for proof.

This proposition, again, motivates our later regression specification (28).

4. Proposition 5 hold qualitatively; that is, if the human capital of an entertainer is large enough, she will gain on net from a greater scope of operation. However, the exact conditions for this proposition will change since M, P and k will be ruled by a different profile of equilibrium conditions. Intuitively, both Propositions 4 and 5 are driven by the fact that entertainers with higher human capital – who therefore earn more – gain more from a capacity enlargement, again because they are able to charge higher prices as they provide higher valued services.
5. Proposition 6 holds qualitatively – namely, an increase in A induces more agents to seek work employing their human capital – though the exact condition may change. For the Cobb-Douglas case where $\widehat{\rho} = 0$ this is true and hence it is also true if $\widehat{\rho}$ is close enough to 0. The intuition in the Cobb-Douglas case is the following. First, recall equation (21), which determines equilibrium k via market clearing of the subsistence good in units of the revenue of the marginal entertainer. Given k, an increase in A increases the supply of the good, while the aggregate demand, measured in units of the marginal entertainer's revenue, is unchanged in the the Cobb-Douglas case because both the proportion of aggregate income to the marginal entertainer's income and the fraction of the aggregate income spent on the subsistence good are unchanged in this case. Therefore, an increase in A must reduce the number of agents who supply labor; that is, moves k leftward.
6. Proposition 7 still holds, namely relatively more talented (and thus richer) entertainers gain relatively more from an increase in A. Again it is driven by the same effect: an increase in A affects entertainers' income by raising aggregate income, and a bigger fraction of this increase accrues to an entertainer with higher human capital because she acquires a bigger fraction of the aggregate spending on entertainment services.

5. Empirical Patterns

In this section we exploit U.S. occupational data over the last three decades to explore key predictions of the model. Here we focus exclusively on testing predictions related to changes in the IRS limit, denoted B in the model. In doing so, we set aside predictions with respect to changes in the productivity of unskilled labor, denoted A in the model, since it is difficult to construct an accurate measure of A across countries and even more difficult to find plausibly exogenous variation associated with it.

To begin, we document key stylized facts associated with the evolution of inequality both within and between occupations. In a regression approach we then formally test Propositions 2 and 4 from the model, exploiting the advent of the internet as a natural experiment. To reiterate, Proposition 2 states that a rise in the IRS limit leads to a fall in employment in "affected" occupations, a notion whose empirical counterpart we define clearly below. Proposition 4 then states that the higher the income of a worker, the more the worker gains (or the less she loses) with a rise in the IRS limit – formally, the log wage gap between workers within affected occupations rises.

We focus on the U.S. in order to exploit detailed annual data on workers' hours and earnings. Specifically, we use data on wages and employment within U.S. occupations from the U.S. Current Population Survey (CPS) over the years 1985 to 2006.[24] We deal with top-coding in the manner described by Bakija, Cole and Heim (2010), though our results are also robust to excluding top earners. Consistent with the literature, we restrict the sample to full-time workers between 18 and 65. Our unit of interest is the occupation, and we adopt a consistent definition of occupations across datasets using the definitions from Autor and Dorn (2013).

We begin by documenting three facts. These are: 40 percent of the rise in aggregate wage inequality over the period 1990-2010 occurred *within* occupations; nearly two-thirds of the rise in within-occupation inequality can be explained by the occupations that were most exposed (top ten percent) to rising internet sales; these most-exposed occupations saw significantly slower employment growth relative to other occupations, on average, over the period.

These facts – formally documented below – indicate that rising within-occupation inequality has been important and, at the same time, can be explained by the subset of occupations that has been most impacted by innovations in ICT. Importantly, the first two facts imply that over a quarter of the recent rise in *aggregate* wage inequality can be explained by these types of occupations, and yet the facts cannot be jointly explained by existing models of technological change. For instance, consider travel agents, whose share of U.S. employment has halved over the past two decades (consistent with Fact 3), even as the average travel agent has seen a 14 percent increase in their real wage over the period and "superstar" travel agents have thrived (Fact 1),[25] an outcome overwhelmingly achieved by exploiting internet-based sales strategies (Fact 2).[26]

5.1. Decomposition of Wage Dispersion

We begin by decomposing aggregate log wage dispersion into within- and between-occupation components separately for 1990, 2000 and 2010. Formally, we calculate:

[24]The data were obtained from IPUMS (see Ruggles, Alexander, Genadek, Goeken, Schroeder and Sobek (2010)).
[25]Source for employment and wage statistics: Bureau of Labor Statistics, Occupational Employment Statistics Survey, 1997 and 2016
[26]For instance, see "How the Internet Created the Superstar Travel Agent": https://www.forbes.com/sites/douggollan/2015/03/27/how-the-internet-created-the-superstar-travel-agent-2

	Overall			Most vs. Least Exposed to Internet Sales			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Year	Total Wage Dispersion	Within-Occupation	Between-Occupation	Within (Most Exposed)	Within (Least Exposed)	Between (Most Exposed)	Between (Least Exposed)
1990	0.298	0.223	0.075	0.028	0.195	0.013	0.062
2000	0.322	0.233	0.089	0.032	0.200	0.014	0.075
2010	0.377	0.254	0.123	0.048	0.206	0.022	0.101

Table 1: Wage Dispersion Within and Between Occupations, 1990-2010

$$\frac{1}{N_t}\sum_i (w_{it} - \bar{w}_t)^2 = \frac{1}{N_t}\sum_l \sum_{i \in l} (w_{it} - \bar{w}_{lt})^2 + \frac{1}{N_t}\sum_l N_{lt}(\bar{w}_{lt} - \bar{w}_t)^2 \tag{26}$$

where workers are indexed by i and the year by t; l represents occupations; N_{lt} and N_t represent the number of workers in each occupation and overall; and w_{it}, $\bar{w}_{lt}$ and $\bar{w}_t$ are the log worker wage, the average log occupational wage, and the overall average wage. In using the log wage we ensure the values are independent of the wage units. The first term on the right hand side reflects the within-occupation component of wage inequality.

Table 1 reports the results. The first three columns of the table highlight the overall contributions of within- and between-occupation inequality. First, throughout the period the contribution of within-occupation inequality to aggregate inequality in any particular year is large relative to the between component. Furthermore, 40 percent of the rise in aggregate inequality between 1990 and 2010 was due to a rise in within-occupation inequality (Fact 1). We can decompose total log wage dispersion further by noting that the within term in (26) is the sum across individual occupations, and so the contribution of different subsets of occupations can be easily separated out. As it turns out, most of the rise in within-occupation inequality was due to a particular subset of occupations, namely those most affected by the internet. To see this, we construct a measure of the extent to which the output of each of 330 U.S. occupations was associated with internet sales in 2000 and 2010, setting 1990 to zero for all occupations. We denote the measure as B^{int} in order to link it conceptually to the market size measure discussed in the Introduction and that will be a focus of the theoretical section. We define the measure in the following way:

$$B^{int}_{it} = \sum_j \left(IntShr_{jt} \times OccShr_{ij,1990} \right) \tag{27}$$

where $IntShr_{jt}$ is the share of industry j sales in year t that was made over the internet and

	Top 10		Bottom 10
1	Financial services sales occupations	**327**	Legislators
2	Motion picture projectionists	**328**	Clergy and religious workers
3	Cabinetmakers and bench carpenters	**329**	Inspectors of agricultural products
4	Editors and reporters	**330**	Welfare service aides
5	Furniture and wood finishers	**331**	Postmasters and mail superintendents
6	Typesetters and Compositors	**332**	Meter readers
7	Other financial specialists	**333**	Mail and paper handlers
8	Broadcast equipment operators	**334**	Hotel clerks
9	Computer Software Developers	**335**	Judges
10	Actors, directors, producers	**336**	Sheriffs, bailiffs, correctional institution officers

Table 2: Top and Bottom 10 Occupations by Exposure to Internet Sales

$OccShr_{ij,1990}$ is the share of occupation i's total hours employed in industry j in 1990.[27] Thus, the latter term reflects the importance of each industry, in terms of labor hours, to each occupation in a period in which the internet was absent, where we use a pre-period occupational structure in order to avoid incorporating effects due to endogenous changes in the composition of occupations caused by the internet. The former term then captures the extent to which firms within each industry sell their output over the internet.[28] Table 2 lists the top 10 (left column) and bottom 10 (right column) occupations in terms of their exposure to internet sales according to this measure.

The last four columns of Table 1 once again decompose total log wage dispersion in each year into the within and between components, but then decompose each of these into two further sets of occupations reflecting 1) the top 10 percent of occupations according to measure (27) and 2) all other occupations. Comparing columns (2) and (4), we see that 65 percent of the rise in within-occupation wage inequality between 1990 and 2010 is due to the set of occupations that were most exposed to internet sales (Fact 2). The stylized facts are collectively depicted in Figure 8.

Again, rising within-occupation inequality has been important and, at the same time, can be explained by the subset of occupations that has been most impacted by innovations in ICT.

Finally, employment growth among the top ten percent of occupations most linked to internet sales was 7 percent over the period, rising from about 7 to 7.5 million workers, significantly slower than the 36 percent average employment growth associated with other occupations (Fact 3).

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[27] Internet sales by industry come from Census' E-Stats database, available at http://www.census.gov/econ/estats/. See Appendix B for more details regarding the construction of the measure.

[28] Of course, the measure may not perfectly capture the extent to which occupational services are linked to internet sales. For instance, even within an industry that sells a substantial amount over the internet, some occupations may be specialized in brick-and-mortar sales, while others are focused on internet sales. Furthermore, our analysis below will focus in part on the implications for wages, but the elasticity of occupational wages to internet sales may vary across occupations for many reasons, from which we abstract.

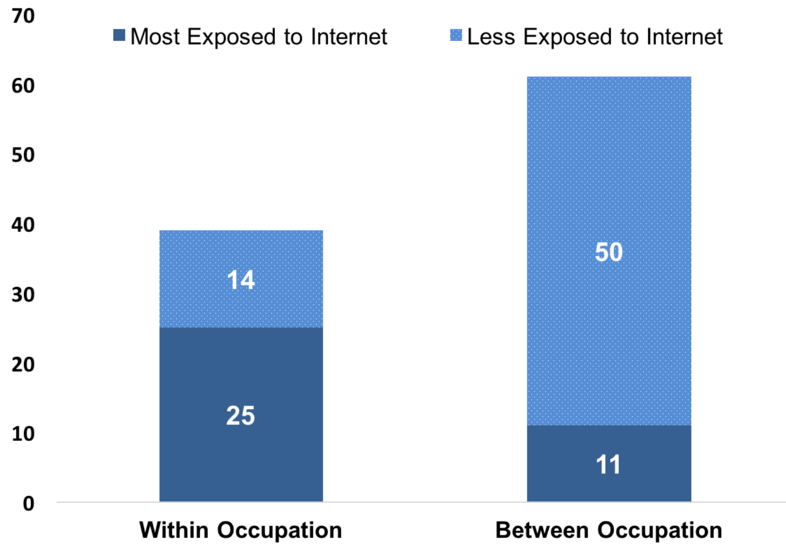

Figure 8: Contributions to Growth in U.S. Wage Inequality, 1990-2010

5.2. Regression Approach

In this section we provide further evidence regarding the impact of the internet on within-occupation wage dispersion. To do this we exploit the natural experiment generated by the growth in use of the internet in the mid-1990s, which differentially exposed workers to a rise in market access for their services over subsequent years. Due to the internet, some workers, such as actors and writers, now face a potential market size that is many millions of consumers larger than it was in 1990, while others, such as bus drivers, face approximately the same set of potential consumers as they did in 1990.

We exploit this differential exposure to potential sales over the internet by estimating the following specification:

$$\triangle WageGap_{i,t:t-1} = c + \beta_1 \left(\triangle B^{int}_{i,t:t-1} \right) + \beta_2 CompUse_{i,1989} + WageGap_{i,80-90} + \epsilon_{it} \tag{28}$$

where we follow recent convention by defining the change in wage inequality, $\triangle WageGap_{i,t:t-1}$, as the change in the gap between the 90th and 10th percentiles of the log wage distribution for each occupation i or, alternatively, the 90-50 log wage gap, where we stack changes over the periods 1990 to 2000 and 2000 to 2010. Importantly, the wage variation we exploit has been cleaned of variation due to age, age squared, sex and level of education in a "first stage" regression; in other words, we effectively control for changes in the composition of workers within occupations. We also clean the wage of industry variation (i.e., include industry fixed effects in the first stage), thereby

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Δ 90-10 Log Wage Gap			**Δ 90-50 Log Wage Gap**			**Δ Log Employment**		
$\Delta InternetSales$	0.354***	0.315*	0.270*	0.142**	0.113**	0.091	-0.023***	-0.021**	-0.018**
	(0.155)	(0.167)	(0.141)	(0.062)	(0.045)	(0.073)	(0.007)	(0.010)	(0.008)
$CompUse_{1989}$		0.064***	0.063***		0.026**	0.024*		0.044*	0.051*
		(0.018)	(0.018)		(0.013)	(0.014)		(0.022)	(0.028)
$WageGap^{90-10}_{1980-1990}$			-0.225***						-0.356***
			(0.084)						(0.061)
$WageGap^{90-50}_{1980-1990}$						-0.073			
						(0.101)			
Observations	642	584	584	642	584	584	642	584	584

Notes: Dependent variables are the stacked change in the 90-10 log wage gap, 90-50 log wage gap and log employment over 1990-2010. In a first-stage regression, wage variation is cleaned of variation in age, age squared, sex, education, and industry. Exposure to internet sales at the occupation level is defined as described in Section 3.1. Computer use in 1989 is obtained from the computer use supplement of the 1989 Current Population Survey. The wage gap controls are the pre-period changes in the wage gap (between 1980 and 1990) for the 90-10 and 90-50 gaps, respectively. Standard errors are in parentheses and are clustered at the occupation level.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 3: Impact of the Rise in Internet Sales on Within-Occupation Inequality, 1990-2010

controlling for differences across industries in the evolution of the wage structure. In addition, in some specifications we control for differential pre-period trends (1980 to 1990) in the wage gap, denoted $WageGap_{i,80-90}$. Standard errors are clustered at the occupation level.

In our strictest specifications we also control for the differential use of computers across occupations in 1989 ($CompUse_{i,1989}$), several years prior to the spread of the internet. Specifically, we control for the share of hours worked in an occupation by workers who use a computer in order to address the possibility that computer use is a key omitted variable in the specification.[29] In other words, the relationship between the change in internet sales within an occupation and rising inequality may be due to the direct impact of increased computer use on both wage inequality and rising internet sales. By controlling for computer use in 1989 we absorb variation in the prevalence of computer use across occupations that is unrelated to (future) internet sales.[30]

Noting that the results are suggestive, and not definitive, Table 3 presents the estimates. In columns (1)-(6) we see that rising internet sales are associated with a widening wage gap, both across the 90-10 distribution as well as the 90-50 distribution. Aside from a lack of statistical significance in column (6), these effects hold across specifications, indicating that the effects are not solely driven by predicted computer use or pre-trends in the wage distribution. The economic magnitudes are also important: the estimates imply that the increase in occupational exposure to

[29]We obtain these data from the Current Population Survey computer use supplement, 1989.

[30]The shortcoming of this control is that it likely does not perfectly predict future computer use in an occupation, and the unexplained portion of future computer use may be correlated with both rising wage inequality and internet sales. At the same time we note that some of this additional variation will likely be absorbed by the pre-trends in the wage gap.

internet sales over the period explains 39 percent of the rise in the 90-10 wage gap and 26 percent of the rise in the 90-50 gap.[31]

In columns (7), (8) and (9) we estimate a specification identical to (28) except with the (stacked) change in log hours worked as the dependent variable.[32] Here we find that increased internet sales are associated with an absolute decline in occupational employment. We note that this is an even stronger result than suggested by the descriptive facts, which showed slow, but positive, growth for the subset of occupations most impacted by internet sales. Since employment growth was positive, on average, over the period – i.e., the effect due to increased exposure to internet sales *offset* overall employment growth – we note that the negative effect estimated here is on the order of 10 percent of the observed, positive employment growth.

In summary, the results indicate that the rapid rise in market access due to the spread of the internet has been associated with an increase in within-occupation inequality, as well as declining employment for the most impacted occupations. In the next section we present a model that can explain these dynamics.

6. Concluding Remarks

New technologies are constantly reshaping economies, generating winners and losers. In this paper we distinguish between two types of technological changes and consider their effects on both the income distribution and occupational choice within a unified model. The first type of technological change increases the quantity of output per unit of time, while the second type reflects a key feature of ICT, namely that it increases the scale of operation for workers in some occupations. This enables a given quantity of output to be used or consumed to a greater extent. We find that both types of technological advancement increase within-occupation inequality, though they do so via different mechanisms. Specifically, the first type of technological change leads to rising inequality via a general equilibrium effect, whereas the second (reflecting ICT advancement) does so by increasing competition between workers, which drives workforce reallocation and redistributes revenue across the workers. Interestingly, we find that these two types of technological changes have opposite implications for occupational choice. The first type encourages a greater number of workers to specialize in human-capital-intensive work and supply differentiated goods or services. In contrast, the second type, by intensifying competitive between workers, drives workers out of the human-capital-intensive occupations.

[31] The mean of the internet exposure measure rose from 0 to 0.05 over the period. Multiplied by the coefficient in column (3) from Table 3 gives a value of 0.014, which is 39 percent of the total rise in the 90-10 wage gap over the period. A similar calculation leads to the value with respect to the 90-50 gap.

[32] In this case we control for the pre-trend in the 90-10 wage gap, but the results are nearly identical to controlling for the pre-trend in the 90-50 gap.

We conclude by noting that there are clearly many other types of technological changes, and each may have different implications for the labor market. Furthermore, there are a range of forces, both technological and otherwise, that have contributed to the rising inequality observed in many countries in recent decades. We believe that the technological forces that we consider here are important in part due to their near ubiquity, as well as the fact that they may be relatively difficult for policy-makers to counter compared to institutional factors, such as the extent of unionization or tax policies.

Appendix A. Proofs

Appendix A.1. Proof of Proposition 2

Proof. k is determined by equation (22). Differentiating with respect to B on both sides, we find

$[d(\mu^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}})/dk - (A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}] \cdot dk/dB = (k-1)d(A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}/dB + d[(1-k_0)(A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}]/dB$

We further know that $d(^{\frac{1}{1-\hat{\rho}}} H_k^{\frac{\rho-\hat{\rho}}{1-\hat{\rho}}} h_k^{\frac{-\rho}{1-\hat{\rho}}})/dk < 0$ because $dH_k/dk < 0$ and $\frac{\rho-\hat{\rho}}{1-\hat{\rho}} > 0$, and $dh_k/dk > 0$. Therefore, on the LHS of the equation the term in front of dk/dB is negative.

On the RHS we consider two cases depending on the sign of $\hat{\rho}$. (i) if $\hat{\rho} \geq 0$, on its RHS, $d(A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}/dB > 0$ and $k-1<0$. Therefore, if

$d[(1-k_0)(A/B+c)\frac{-\hat{\rho}}{1-\hat{\rho}}]/dB \leq 0$,

which (as $k_0 = \frac{Bc}{A+Bc}$) is equivalent to $\hat{\rho} \leq \frac{Bc}{A+Bc}$, then the RHS is negative and thus $dk/dB > 0$.

(ii) if $\hat{\rho} < 0$, note that the RHS of the equation that pins down dk/dB (see the second line of the proof above) is equal to $(k-k_0) \times \frac{d(A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}}{dB} + (A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}} \times \frac{d(1-k_0)}{dB}$. If $\hat{\rho} < 0$, then $\frac{-\hat{\rho}}{1-\hat{\rho}} > 0$ and hence $\frac{d(A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}}{dB} < 0$. Given $k > k_0$, we then have $(k-k_0) \times \frac{d(A/B+c)^{\frac{-\hat{\rho}}{1-\hat{\rho}}}}{dB} < 0$. Moreover, with $1-k_0 = \frac{A}{A+Bc}$, obviously $\frac{d(1-k_0)}{dB} < 0$. Therefore, the RHS of (22) is negative and thus $dk/dB > 0$.. ∎

Appendix A.2. Proof of Proposition 3

Proof. We need only show that $dm_i/dB < 0$ for $i = k$. When this is the case, the Proposition follows from the fact that dm_i/dB is continuous in i. By (16), $\frac{dm_i}{dB} = h_i^\rho/h_k^\rho \cdot [c-(A+Bc)\cdot\rho\cdot(\log h_k)'\cdot\frac{dk}{dB}]-c$. At $i = k$, therefore, $\frac{dm_i}{dB} = c-(A+Bc)\cdot\rho\cdot(\log h_k)'\cdot\frac{dk}{dB} - c = -(A+Bc)\cdot\rho\cdot(\log h_k)'\cdot\frac{dk}{dB} < 0$ because $(\log h_k)'$ is assumed to be positive and $\frac{dk}{dB} > 0$ by Proposition 2. ∎

Appendix A.3. Proof of Proposition 4

Proof. We go to prove that $\frac{d\log m_i}{dB}$ increases with m_i. Let $\widetilde{m}_i := m_i + Bc$. Then

$$\frac{d\log m_i}{dB} = \frac{d\log \widetilde{m}_i}{dB} \times \frac{\widetilde{m}_i}{m_i} - \frac{c}{m_i}. \tag{A.1}$$

By (16

$$\widetilde{m}_i = (Bc+A)\frac{h_i^\rho}{h_k^\rho}.$$

Thus $\log \widetilde{m}_i = \log(Bc+A) + \rho\log h_i - \rho\log h_k$ and

$$\frac{d\log \widetilde{m}_i}{dB} = \frac{c}{Bc+A} - \rho\frac{h_k'}{h_k}\frac{dk}{dB} \tag{A.2}$$

and is independent of m_i. Hence, from (A.1), $\left[\frac{d\log m_i}{dB}\right]'_{m_i} = \frac{d\log \widetilde{m}_i}{dB} \times \left[\frac{\widetilde{m}_i}{m_i}\right]'_{m_i} - \left[\frac{c}{m_i}\right]'_{m_i} = \frac{d\log \widetilde{m}_i}{dB} \times \frac{-Bc}{m_i^2} + \frac{c}{m_i^2}$. It follows that $\left[\frac{d\log m_i}{dB}\right]'_{m_i} > 0$ if and only if

$$\begin{aligned} \frac{d\log \widetilde{m}_i}{dB} \times \frac{-Bc}{m_i^2} + \frac{c}{m_i^2} &> 0 \Leftrightarrow \\ -\frac{d\log \widetilde{m}_i}{dB} \times B + 1 &> 0, \end{aligned}$$

which, from (A.2), is equivalent to

$$1 > \left[\frac{c}{Bc + A} - \rho \frac{h'_k}{h_k}\frac{dk}{dB}\right] \times B.$$

This inequality, as $h'_k > 0$ and $\frac{dk}{dB} > 0$ (Proposition 2), follows from $1 > \frac{c}{Bc+A} \times B$, which obviously holds true. ∎

Appendix A.4. Proof of Lemma 1

Proof. By (16),

$$\frac{dm_i}{dB} = h_i^\rho/h_k^\rho \cdot [c - (A + Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB}] - c. \tag{A.3}$$

The identity of the marginal entertainer, k, is determined by equation (22). Taking the logarithm of both sides: $\frac{1}{1-\widehat{\rho}} \log \mu + \frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})} \log H_k^\rho - \frac{\rho}{1-\widehat{\rho}} \log h_k = \log(k - k_0) - \frac{\widehat{\rho}}{1-\widehat{\rho}} \log(A/B + c)$. Now taking the derivative with respect to B on both sides and noting that $\frac{dH_k^\rho}{dk} = -h_k^\rho$ and recalling $k_0 = \frac{Bc}{A+Bc}$: $[-\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})} h_k^\rho/H_k^\rho - \frac{\rho}{1-\widehat{\rho}}(\log h_k)'] \cdot \frac{dk}{dB} = \frac{1}{k-k_0} \cdot [\frac{dk}{dB} - \frac{Ac}{(A+Bc)^2}] + \frac{\widehat{\rho}}{1-\widehat{\rho}} \cdot \frac{A}{(A+Bc)B} \Rightarrow$

$$\frac{dk}{dB} = \frac{1/(k - k_0) \cdot Ac/(A + Bc)^2 - \widehat{\rho}/(1 - \widehat{\rho}) \cdot A/[A + Bc)B]}{1/(k - k_0) + \frac{\rho}{1-\widehat{\rho}}(\log h_k)' + \frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})} h_k^\rho/H_k^\rho}.$$

The numerator is smaller than $1/(k - k_0) \cdot Ac/(A + Bc)^2$, while the denominator is greater than $1/(k - k_0) + \frac{\rho}{1-\widehat{\rho}}(\log h_k)'$, which is in turn greater than $1/(k - k_0) + \rho(\log h_k)'$. Therefore,

$$\frac{dk}{dB} < \frac{Ac/(A + Bc)^2}{1 + \rho(\log h_k)'(k - k_0)}. \tag{A.4}$$

By (A.3), $\frac{dm_i}{dB} > 0$ if

$$h_i^\rho/h_k^\rho \cdot [c - (A + Bc) \cdot \rho \cdot (\log h_k)' \cdot \frac{dk}{dB}] > c. \tag{A.5}$$

With an upper bound of $\frac{dk}{dB}$ given by (A.4), this inequality follows from: $h_i^\rho/h_k^\rho \cdot [c - (A + Bc) \cdot \rho \cdot$

$(\log h_k)' \cdot \frac{Ac/(A+Bc)^2}{1+\rho(\log h_k)'(k-k_0)}] > c \Leftrightarrow$

$$h_i^\rho/h_k^\rho \cdot [1 - \frac{A}{A+Bc} \cdot \frac{\rho \cdot (\log h_k)'}{1+\rho(\log h_k)'(k-k_0)}] > 1, \tag{A.6}$$

which is equivalent to (24). ∎

Appendix A.5. Proof of Lemma 2

Proof. We prove the lemma in three steps.

Step 1: If $h_1 > 1$, then

$$k - k_0 < D(\frac{h_1}{h_k})^{\frac{\rho}{1-\widehat{\rho}}}(1-k)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}}. \tag{A.7}$$

Proof: k is determined by equation (22), or equivalently, $k - k_0 = DH_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}} h_k^{\frac{-\rho}{1-\widehat{\rho}}}$. Note that $H_k = \{\int_k^1 h_i^\rho\}^{\frac{1}{\rho}}|_{h_i'>0} < \{\int_k^1 h_1^\rho\}^{\frac{1}{\rho}} = h_1(1-k)^{\frac{1}{\rho}}$. Therefore, $H_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}} h_k^{\frac{-\rho}{1-\widehat{\rho}}} = (\frac{H_k^{\rho-\widehat{\rho}}}{h_k^\rho})^{\frac{1}{1-\widehat{\rho}}} < (\frac{h_1^{\rho-\widehat{\rho}}(1-k)^{\frac{\rho-\widehat{\rho}}{\rho}}}{h_k^\rho})^{\frac{1}{1-\widehat{\rho}}} = h_1^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}}/h_k^{\frac{\rho}{1-\widehat{\rho}}} \cdot (1-k)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}}|_{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}} \le \frac{\rho}{1-\widehat{\rho}} \text{ and } h_1>1} < h_1^{\frac{\rho}{1-\widehat{\rho}}}/h_k^{\frac{\rho}{1-\widehat{\rho}}} \cdot (1-k)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}}$, which implies (A.7).

Step 2:

$$k < f(k_0, D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\widehat{\rho}}}). \tag{A.8}$$

Proof: Let $\tau := f(k_0, D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\widehat{\rho}}})$. By the definition of $f(\cdot,\cdot)$, $\tau - k_0 = D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\widehat{\rho}}} \cdot (1-\tau)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}}$. The two sides of this inequality minus, respectively, the two sides of inequality (A.7) leads to $\tau - k > D(\frac{h_1}{h_k})^{\frac{\rho}{1-\widehat{\rho}}}[(1-\tau)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}} - (1-k)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}}]$. This inequality can hold true only if $\tau > k$: if $\tau \le k$, then the LHS of the inequality is negative, while the RHS is positive – and thus cannot be strictly smaller than the LHS – because $1 - \tau \ge 1 - k$, which implies $(1-\tau)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}} - (1-k)^{\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})}} \ge 0$ (as $\frac{\rho-\widehat{\rho}}{\rho(1-\widehat{\rho})} > 0$). Q.E.D.

Step 3: We prove the Lemma by showing that $\zeta \ge h_1/h_k$ leads to a contradiction. Clearly, $f(k_0, y)$ increases with y, and therefore if $\zeta \ge h_1/h_k$, then $f(k_0, D \cdot (\frac{h_1}{h_k})^{\frac{\rho}{1-\widehat{\rho}}}) < (f(k_0, D \cdot \zeta^{\frac{\rho}{1-\widehat{\rho}}})$, which together with (A.8) implies that $k < f(k_0, D \cdot \zeta^{\frac{\rho}{1-\widehat{\rho}}})$. Since $h'(i) > 0$, then $h_k < h(\ f(k_0, D \cdot \zeta^{\frac{\rho}{1-\widehat{\rho}}}))$. Thus we have

$$\zeta \ge \frac{h_1}{h_k} > \frac{h_1}{h(f(k_0, D \cdot \zeta^{\frac{\rho}{1-\widehat{\rho}}}))},$$

which implies $\zeta \cdot h(f(k_0, D \cdot \zeta^{\frac{\rho}{1-\widehat{\rho}}})) > h_1$, in contradiction to the lemma. Q.E.D. ∎

Appendix A.6. Proof of Proposition 6

Proof. k is determined by equation (22). Differentiate with respect to A on both sides, and we find

$$[d(\mu^{\frac{1}{1-\widehat{\rho}}} H_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}} h_k^{\frac{-\rho}{1-\widehat{\rho}}})/dk - (A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}}] \cdot dk/dA = (k-1)d(A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}}/dA + d[(1-k_0)(A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}}]/dA$$

We saw $d(^{\frac{1}{1-\widehat{\rho}}} H_k^{\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}}} h_k^{\frac{-\rho}{1-\widehat{\rho}}})/dk < 0$ because $dH_k/dk < 0$ and $\frac{\rho-\widehat{\rho}}{1-\widehat{\rho}} > 0$, and $dh_k/dk > 0$. Therefore, on the LHS of the equation the term in front of dk/dA is negative.

On its RHS, we consider two cases depending on the sign of $\widehat{\rho}$. (i) if $\widehat{\rho} \geq 0$, $d(A/B+c)\frac{-\widehat{\rho}}{1-\widehat{\rho}}/dA < 0$ and $k-1 < 0$. Therefore, if

$d[(1-k_0)(A/B+c)\frac{-\widehat{\rho}}{1-\widehat{\rho}}]/dA \geq 0$,

which (as $k_0 = \frac{Bc}{A+Bc}$) is equivalent to $\widehat{\rho} \leq \frac{Bc}{A+Bc}$, then the RHS is positive and

thus $dk/dA < 0$.

(ii) if $\widehat{\rho} < 0$, note that the RHS of the equation that pins down dk/dB (probably we shall number of the equation) is equal to $(k-k_0) \times \frac{d(A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}}}{dA} + (A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}} \times \frac{d(1-k_0)}{dA}$. If $\widehat{\rho} < 0$, then $\frac{-\widehat{\rho}}{1-\widehat{\rho}} > 0$ and hence $\frac{d(A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}}}{dA} > 0$. Given $k > k_0$, we then have $(k-k_0) \times \frac{d(A/B+c)^{\frac{-\widehat{\rho}}{1-\widehat{\rho}}}}{dA} > 0$. Moreover, with $k_0 = \frac{Bc}{A+Bc}$, obviously $\frac{d(1-k_0)}{dA} = -\frac{dk_0}{dA} > 0$. Therefore, the RHS of (22) is positive and thus $dk/dA < 0$. ∎

Appendix A.7. Proof of Proposition 7

Proof. By (16), $\frac{dm_i}{dA} = \frac{h_i^\rho}{h_k^\rho} + (Bc+A)(-\rho)\frac{h_i^\rho}{h_k^{\rho+1}} \cdot h_k' \cdot \frac{dk}{dA} = \frac{h_i^\rho}{h_k^\rho} \cdot [1+(Bc+A)(\log h_k)' \cdot (-\rho\frac{dk}{dA})]|_{-\rho\frac{dk}{dA}>0 \text{ (by Prop. 5)}} > \frac{h_i^\rho}{h_k^\rho} \geq 1$. Moreover, by (16), $\frac{h_i^\rho}{h_k^\rho} = \frac{m_i+Bc}{A+Bc}$. Then, $\frac{dm_i}{dA} = \frac{m_i+Bc}{A+Bc} \cdot [1+(Bc+A)(\log h_k)' \cdot (-\rho\frac{dk}{dA})]$ and increases with m_i. ∎

Appendix A.8. Proof of Proposition 8

Proof. For entertainers $i < i^*$, their capacity constraint is non-binding and the optimal price for them is thus $p_i = c/\rho$. From equation (6) and (16), $m_i = \varpi(B) \times h_i^{\rho/(1-\rho)}$ for some function of B, $\varpi(B)$. Thus $\frac{d\log m_i}{dB} = \frac{d\log\varpi(B)}{dB}$ is independent of m_i or weakly increasing with it. For entertainers $i > i^*$, their capacity constraint is binding. Thus equation (13) holds. Following the discussion ensuing this equation we find that $p_i = p_{i^*} \times h_i^\rho/h_{i^*}^\rho$ and hence $m_i = (p_i - c)B = p_{i^*}B \times h_i^\rho/h_{i^*}^\rho - Bc$. As the capacity constraint just starts binding at i^*, we know that $p_{i^*} = c/\rho$ and independent of B. Furthermore, obviously $di^*/dB > 0$, that is a larger B reduces the number of the agents to whom the capacity constraint is binding. Then following the proof of Propositions 4 we let $\widetilde{m}_i := m_i + Bc = p_{i^*}B \times h_i^\rho/h_{i^*}^\rho$. And we have $\frac{d\log\widetilde{m}_i}{dB} = \frac{1}{B} - \rho\frac{h_{i^*}'}{h_{i^*}}\frac{di^*}{dB}$ and that $\left[\frac{d\log m_i}{dB}\right]_{m_i}' > 0 \Leftrightarrow -\frac{d\log\widetilde{m}_i}{dB} \times B + 1 > 0 \Leftrightarrow -\left[\frac{1}{B} - \rho\frac{h_{i^*}'}{h_{i^*}}\frac{di^*}{dB}\right] \times B + 1 > 0 \Leftrightarrow \rho\frac{h_{i^*}'}{h_{i^*}}\frac{di^*}{dB} \times B > 0$, which holds true as we saw $\frac{di^*}{dB} > 0$. ∎

Appendix B. Internet Exposure Measure

Our measure of internet exposure, B_{it}^{Int}, is constructed as in (27). The industry internet sales data come from Census' E-Stats database, which provides the data at the two- and three-digit North American Industry Classification System (NAICS) level. We then concord these to the Ind1990 classification used in the CPS using a straightforward concordance provided by Census. One nuance is that some of the sales data is classified under the industry "E-Merchants" (NAICS 4541) by product, in categories such as Books and Magazines, Music and Videos, etc. We therefore match these to the relevant Ind1990 industries manually. The final step is to calculate (27).

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